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Regular Polygonal and Prismatic Tverberg-Type Theorems

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The celebrated Tverberg Theorem, generalizing Radon's Theorem when q = 2, states that any T(q,d) = (q-1)(d+1)+1 points in R^d can be partitioned into q pairwise disjoint sets whose convex hulls have non-empty q-fold intersection. Moreover, this “Tverberg number” T(q,d) is generically tight. We will show that in the absence of a full Tverberg partition for fewer than T(q,d) points, one can nonetheless guarantee a partition of these points into q pairwise disjoint sets so there are q points, one from each of the resulting convex hulls, which form the vertices of a regular q-gon. Analogous results hold for regular prisms when q is composite. As with Tverberg's original theorem, these results can be extended to the continuous setting when q is a prime power, and these likewise admit constrained versions: restrictions on the number of points in each of the disjoint sets (van-Kampen–Flores type results) and the prescription of equal distance from each vertex of the regular polygon to the original set, ``balanced” colored versions of a Sobéron variety, and so on.
