Louis Billera (Cornell)

Balanced & Unbalanced Collections and the Real Linear Algebra of 0/1-Vectors

Abstract for the Combinatorics Seminar 2018 October 30

Collections of proper subsets of an n-set come in two flavors, balanced or unbalanced, according to whether, when considered as sets of vertices of the n-cube, their convex hull meets or misses the main diagonal. The former arose more than 50 years ago in the context of economic equilibria, the latter more recently in quantum physics. In each case, it is of interest to enumerate minimal (or maximal, as appropriate) such collections.

Pursuit of this problem leads directly to the need to understand the real linear algebra (i.e., the real matroid) of the set of all 0/1-vectors. It shouldn't be surprising that a question this basic would come from a variety of applications. It also lends itself to a variety of possible solution techniques, involving combinatorics, geometry, topology, algebra and even number theory. Asking a bit more gets you back to an 1893 question of Hadamard and to the hypercomplex numbers (quaternions, octonions, and beyond) constructed a half-century earlier. I will describe some of these approaches in the hope of spurring interest in these questions.
