Boris Bukh (Carnegie Mellon)

Topological Version of Pach's Overlap Theorem

Abstract for the Combinatorics and Geometry/Topology Seminars 2018 May 1

Consider the collection of all the simplices spanned by some n-point set in $\mathbf{R}^d$. There are several results showing that simplices defined in this way must overlap very much. In this talk I focus on the generalization of these results to 'curvy' simplices.

Specifically, Pach showed that every d+1 sets of points, Q_1, ..., Q_{d+1}, in $\mathbf{R}^d$ contain linearly-sized subsets P_i in Q_i such that all the transversal simplices that they span intersect. In joint work with Alfredo Hubard, we show, by means of an example, that a topological extension of Pach's theorem does not hold with subsets of size $C(\log n)^{1/(d-1)}$. We show that this is tight in dimension 2, for all surfaces other than S^2. Surprisingly, the optimal bound for S^2 is $(\log n)^{1/2}$. This improves upon results of Barany, Meshulam, Nevo, Tancer.
