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Relaxations of the Matroid Axioms: h-Vectors

Abstract for the Combinatorics Seminar 2015 September 1

Motivated by a question of Duval and Reiner about eigenvalues of combinatorial Laplacians, Jeremy Martin, Ernest Chong, Steven Klee, and I have developed a generalization of (ordered) matroid theory to wider classes of simplicial complexes. In addition to all independence complexes of matroids, each such class contains all pure shifted simplicial complexes, and it retains a little piece of matroidal spirit and structure. To achieve this, we relax the various cryptomorphic definitions of a matroid. In contrast to the matroid setting, these relaxations are independent of each other, i.e., they produce different extensions.

Imposing various combinations of these new axioms allows us to prove analogues of many classical matroid structures and properties. Examples of such properties include the Tutte polynomial, shellability of the order complex, the existence of the no-broken-circuit complex and its shellability, the broken circuit basis of the Orlik-Solomon algebra, and many others.

After this I will discuss the h-vectors of complexes that satisfy our relaxed version of the exchange axiom, extend Stanley's pure O-sequence conjecture about the h-vector of a matroid, solve this conjecture for the special case of shifted complexes, and speculate a bit about the general case.
