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Generalized Quadrangles With Symmetry

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A generalized quadrangle is a point-line incidence geometry Q such that (1) any two points lie on at most one line, and (2) given a line l and a point P not incident with l, P is collinear with a unique point of l. Generalized quadrangles are a specific type of generalized polygon, which were first introduced by Tits in 1959 as geometries associated to classical groups. It is natural, then, to ask the question: if one starts with the abstract definition of a generalized quadrangle, which ones are highly symmetric? I will discuss the background of this question, leading to the following recent work, joint with John Bamberg and Cai-Heng Li.

An antiflag of a generalized quadrangle is a non-incident point-line pair (P, l). We say that the generalized quadrangle Q is antiflag-transitive if the group of collineations (automorphisms that send points to points and lines to lines) is transitive on the set of all antiflags. We prove that if a finite, thick generalized quadrangle Q is antiflag-transitive, then Q is one of the following: the unique generalized quadrangle of order (3,5), a classical generalized quadrangle, or a dual of one of these.

The talk will assume no prior knowledge of finite geometry.
