## **Zhongshan Li (Georgia State)**

## Sign Vectors of Subspaces of Rn and Minimum Ranks of Sign Patterns

## Abstract for the Combinatorics Seminar 2014 November 18

A sign pattern (matrix) is a matrix whose entries are from the set  $\{+, -, 0\}$ . The minimum rank of a sign pattern matrix A is the smallest possible rank of a real matrix whose entries have signs indicated by A.

I establish a direct connection between an  $m \times n$  sign pattern with minimum rank  $r \ge 2$  and an m point-n hyperplane configuration in  $\mathbf{R}^{r-1$ . I give a possibly smallest example of a sign pattern (with minimum rank 3) whose minimum rank cannot be realized rationally. For every sign pattern with at most 2 zero entries in each column, the minimum rank can be realized rationally.

Using a new approach involving sign vectors of subspaces and oriented matroid duality, I show that for every  $m \times n$  sign pattern with minimum rank  $\ge n-2$ , rational realization of the minimum rank is possible. Also, for every integer  $n \ge 9$ , there is a positive integer m, such that there exists an  $m \times n$  sign pattern with minimum rank n-3 for which rational realization is not possible.

I give a characterization of m  $\times$  n sign patterns A with minimum rank n - 1, along with a more general description of sign patterns with minimum rank r, in terms of sign vectors of certain subspaces.

I discuss a number of results on the maximum and minimum numbers of sign vectors of k-dimensional subspaces of  $\mathbf{R}^n$ ; this maximum number is equal to the total number of cells of a generic central hyperplane arrangement in  $\mathbf{R}^k$ . For example, the maximum number of sign vectors of a 2-dimensional subspace of  $\mathbf{R}^n$  is 4n + 1 and the maximum number of sign vectors of a 3-dimensional subspace of  $\mathbf{R}^n$  is 4n(n-1) + 3.

Along the way I state related results and open problems.

From

http://www2.math.binghamton.edu/ - **Department of Mathematics and Statistics, Binghamton University** 

Permanent link:

http://www2.math.binghamton.edu/p/seminars/comb/abstract.201411li

Last update: 2020/01/29 19:03