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Zhongshan Li (Georgia State)

Sign Vectors of Subspaces of Rn and Minimum Ranks of Sign Patterns

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A sign pattern (matrix) is a matrix whose entries are from the set $\{+, -, 0\}$. The minimum rank of a sign pattern matrix A is the smallest possible rank of a real matrix whose entries have signs indicated by A.

I establish a direct connection between an $m \times n$ sign pattern with minimum rank $r \ge 2$ and an m point-n hyperplane configuration in $\mathbf{R}^{r-1.1}$ give a possibly smallest example of a sign pattern (with minimum rank 3) whose minimum rank cannot be realized rationally. For every sign pattern with at most 2 zero entries in each column, the minimum rank can be realized rationally.

Using a new approach involving sign vectors of subspaces and oriented matroid duality, I show that for every $m \times n$ sign pattern with minimum rank $\ge n - 2$, rational realization of the minimum rank is possible. Also, for every integer $n \ge 9$, there is a positive integer m, such that there exists an $m \times n$ sign pattern with minimum rank n - 3 for which rational realization is not possible.

I give a characterization of m \times n sign patterns A with minimum rank n - 1, along with a more general description of sign patterns with minimum rank r, in terms of sign vectors of certain subspaces.

I discuss a number of results on the maximum and minimum numbers of sign vectors of k-dimensional subspaces of \mathbf{R}^n ; this maximum number is equal to the total number of cells of a generic central hyperplane arrangement in \mathbf{R}^k . For example, the maximum number of sign vectors of a 2-dimensional subspace of \mathbf{R}^n is 4n + 1 and the maximum number of sign vectors of a 3-dimensional subspace of \mathbf{R}^n is 4n(n-1) + 3.

Along the way I state related results and open problems.

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