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Sign Vectors of Subspaces of \mathbf{R}^n and Minimum Ranks of Sign Patterns

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A sign pattern (matrix) is a matrix whose entries are from the set $\{+, -, 0\}$. The minimum rank of a sign pattern matrix A is the smallest possible rank of a real matrix whose entries have signs indicated by A .

I establish a direct connection between an $m \times n$ sign pattern with minimum rank $r \geq 2$ and an m point- n hyperplane configuration in \mathbf{R}^{r-1} . I give a possibly smallest example of a sign pattern (with minimum rank 3) whose minimum rank cannot be realized rationally. For every sign pattern with at most 2 zero entries in each column, the minimum rank can be realized rationally.

Using a new approach involving sign vectors of subspaces and oriented matroid duality, I show that for every $m \times n$ sign pattern with minimum rank $\geq n - 2$, rational realization of the minimum rank is possible. Also, for every integer $n \geq 9$, there is a positive integer m , such that there exists an $m \times n$ sign pattern with minimum rank $n - 3$ for which rational realization is not possible.

I give a characterization of $m \times n$ sign patterns A with minimum rank $n - 1$, along with a more general description of sign patterns with minimum rank r , in terms of sign vectors of certain subspaces.

I discuss a number of results on the maximum and minimum numbers of sign vectors of k -dimensional subspaces of \mathbf{R}^n ; this maximum number is equal to the total number of cells of a generic central hyperplane arrangement in \mathbf{R}^k . For example, the maximum number of sign vectors of a 2-dimensional subspace of \mathbf{R}^n is $4n + 1$ and the maximum number of sign vectors of a 3-dimensional subspace of \mathbf{R}^n is $4n(n - 1) + 3$.

Along the way I state related results and open problems.

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