Zhongshan Li (Georgia State)

Sign Vectors of Subspaces of Rn and Minimum Ranks of Sign Patterns

Abstract for the Combinatorics Seminar 2014 November 18

A sign pattern (matrix) is a matrix whose entries are from the set {+, −, 0}. The minimum rank of a sign pattern matrix A is the smallest possible rank of a real matrix whose entries have signs indicated by A.

I establish a direct connection between an m × n sign pattern with minimum rank r ≥ 2 and an m point–n hyperplane configuration in $\mathbf{R}^{r-1}$. I give a possibly smallest example of a sign pattern (with minimum rank 3) whose minimum rank cannot be realized rationally. For every sign pattern with at most 2 zero entries in each column, the minimum rank can be realized rationally.

Using a new approach involving sign vectors of subspaces and oriented matroid duality, I show that for every m × n sign pattern with minimum rank ≥ n − 2, rational realization of the minimum rank is possible. Also, for every integer n ≥ 9, there is a positive integer m, such that there exists an m × n sign pattern with minimum rank n − 3 for which rational realization is not possible.

I give a characterization of m × n sign patterns A with minimum rank n − 1, along with a more general description of sign patterns with minimum rank r, in terms of sign vectors of certain subspaces.

I discuss a number of results on the maximum and minimum numbers of sign vectors of k-dimensional subspaces of $\mathbf{R}^n$; this maximum number is equal to the total number of cells of a generic central hyperplane arrangement in $\mathbf{R}^k$. For example, the maximum number of sign vectors of a 2-dimensional subspace of $\mathbf{R}^n$ is 4n + 1 and the maximum number of sign vectors of a 3-dimensional subspace of $\mathbf{R}^n$ is 4n(n − 1) + 3.

Along the way I state related results and open problems.
