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Permutation Descent Statistics via Polyhedral Geometry

Abstract for the Combinatorics Seminar 2014 September 2

Permutations are some of the most fundamental objects of mathematics. A basic combinatorial statistic of a permutation $\pi \in S_n$ is the number of *descents*, des(π) := #{ j : π(j) > π(j+1) }. Euler realized that

$\sum_{k \geq 0} (k+1)^n t^k$ = ($\sum_{\pi \in S_{n}$} $t^{des(\pi)}$) / $(1-t)^{n+1}$

and there have been various generalizations of this identity, most notably when S_n gets replaced by another Coxeter group.

I will illustrate how one can view Euler's identity (and its generalizations) geometrically through enumerating integer points in certain polyhedra. This gives rise to “short” proofs of known theorems, as well as new identities.

This is joint work with Ben Braun (Kentucky).
