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On the Covering Number of Small Symmetric Groups and the Erdös-Ko-Rado Theorem

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Any finite non-cyclic group is the set-theoretic union of finitely many proper subgroups. The minimal number of subgroups needed to cover the group is called the covering number of the group. It is well known that no group is the union of two proper subgroups and it is less well known that already Scorza showed in 1926 that a group is the union of three proper subgroups if and only if it has a homomorphic image isomorphic to the Klein-4-group. The question arises of what is the covering number of a given group and which values n can occur as covering numbers. Tomkinson showed that the covering number of any solvable non-cyclic group has the form prime power plus one and for every number of this form there exists a solvable group with this covering number. He suggested the problem of determining the covering number of nonsolvable groups.

In this talk I will discuss on how to determine the covering number of some small symmetric groups, such as Sym(n) for n = 8, 9, 10, and 12. In addition to some group-theoretic tools we use combinatorics, such as the Erdös-Ko-Rado Theorem, to solve some linear optimization problems. In other cases we have to rely on the computer to solve these problems with the help of GAP and Gurobi.

This is joint work with Daniela Nikolova and Eric Swartz.
