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Manifold Complexity and Face Enumeration

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There are several notions of complexity for three-manifolds. One such definition was introduced by Matveev in the 90's. For a three-manifold M, with or without boundary, his complexity of M is a nonnegative integer c(M). Two properties of c(M) are: (A) For any n there are only finitely many closed, irreducible orientable three-manifolds with c(M) less than n. (B) c(M) is additive with respect to connected sums: c(M#N)=c(M)+c(N).

While there are many combinatorial measures of complexity which satisfy (A), the natural ones to try, such as the minimum number of vertices or tetrahedra in a simplicial triangulation of M, badly fail (B). My goal is to see how c(M) can help understand face enumeration of simplicial triangulations of M.

I will also explore the possibility that there might be simple combinatorial invariants which satisfy (A) and (B) for 3-manifolds, and which have natural extensions to PL-triangulations in higher dimensions which still satisfy (A).

The talk will assume no specialized knowledge of 3-manifolds.
