Rigoberto Flórez (South Carolina at Sumter)

Projective Representation of Non-Representable Matroids (of Biased Graphs)

Abstract for the Combinatorics Seminar 2011 May 23

To each quasigroup Q there is a complete graph K_3 with multiple edges corresponding to the elements of Q, with a class ***B*** of selected triangles such that every two non-parallel edges belong to exactly one selected triangle. This is called a “biased expansion” of K_3, written $Q{\cdot}K_3$. There are two associated rank-3 matroids, the “full frame matroid” $G^*(Q{\cdot}K_3)$ and “extended lift matroid” $L_0(Q{\cdot}K_3)$.

When Q is a subgroup of the multiplicative or additive group of a skew field F, the full frame or extended lift matroid (respectively) is representable in the projective plane over F. Thomas Zaslavsky and I are generalizing this standard theorem to arbitrary quasigroups, the role of F being taken by a planar ternary ring associated with a projective plane. There are complications; for instance, although the skew field associated with a Desarguesian plane is unique, there is not a unique planar ternary ring for a non-Desarguesian plane.
