Samuel Kolins (Cornell)

Simplicial Cell Decompositions of Balls

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A simplicial cell decomposition of a manifold is a generalization of the idea of a triangulation. The faces of a simplicial cell decomposition are still simplexes but two faces can intersect in a subcomplex of their boundaries instead of just a single face. In particular there can be multiple faces on the same vertex set.

The f-vector of a simplicial cell decomposition is the list of the numbers of simplexes of each dimension in the decomposition. I will discuss attempts to characterize the possible f-vectors of simplicial cell decompositions of various manifolds. Complete characterizations are known in the cases of spheres (Stanley '91 and Masuda '05), products of spheres, and real projective spaces (Murai '10). I have recently worked on characterizing the f-vectors of balls. This involves using tools from commutative algebra to obtain new necessary conditions on the f-vectors, as well as some constructive results. I will give a complete characterization of the possible f-vectors of balls in all even dimensions and odd dimensions less than or equal to five, though the problem remains open in higher odd dimensions.
