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Topological Formality of Arrangements of Subspaces Derived from Edge-Colored Hypergraphs

Abstract for the Combinatorics and Geometry and Topology Seminars 2009 October 13

An arrangement of linear hyperplanes in $\mathbf{C}^n$ generates subspaces defined by intersections of hyperplanes. A choice of such subspaces is an *arrangement of subspaces*. When the subspaces are hyperplanes the arrangement's characteristic polynomial carries information about the topology of M, the complement of the subspaces in $\mathbf{C}^n$; this is a fundamental theorem of Orlik and Solomon.

An arrangement of subspaces of the braid arrangement (the arrangement that consists of all hyperplanes with equations $x_i = x_j$) can be encoded by an edge-colored hypergraph. The characteristic polynomial of this type of subspace arrangement is given by a generalized chromatic polynomial of the associated edge-colored hypergraph. However, this polynomial is less informative than in the case of hyperplane arrangements. Stronger topological information about M can be found directly in the hypergraph.

A *Massey product* is an algebraic simplification in cohomology. I will present a sufficient condition for the existence of non-trivial Massey products in the cohomology of M. The condition is proved by studying a spectral sequence associated to the Lie coalgebras of Sinha and Walter. These coalgebras are constructed from the cohomology of M.

If time permits I will construct a family of subspace arrangements whose intersection lattices have the shape of Pascal's triangle. Even though the intersection lattices are not geometric, the complex complements of the arrangements have the property of *rational formality*, i.e., their homotopy type is determined by their rational cohomology.

Everything I will talk about is combinatorial. Some of the topics are in combinatorial topology, but I will try my best to not be overly technical.
