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# On the Covering Number of Symmetric and Alternating Groups: An Exercise in Combinatorial Optimization

## Abstract for the Combinatorics and Algebra Seminars 2009 April 14

Any finite non-cyclic group is the set-theoretic union of finitely many proper subgroups. The minimal number of subgroups needed to cover the group is called the “covering number” of the group. It is well known that no group is the union of two proper subgroups; it is less well known that already in 1926 Scorza showed that a group is the union of three proper subgroups if and only if it has a homomorphic image isomorphic to the Klein-4-group. The questions arise: what is the covering number of a given group, and what values n can occur as covering numbers? Tomkinson showed that the covering number of any solvable non-cyclic group has the form prime power plus one. In addition he showed that there exists no group with covering number 7 and conjectured that there are no groups with covering numbers 11, 13, and 15.

In this context it is of interest to determine the covering numbers of non-solvable and in particular simple groups. So far very little is known; otherwise very accessible groups, like alternating and symmetric groups, pose a particular challenge. We determine the covering numbers of some small alternating and symmetric groups. This is an exercise in combinatorial optimization and some of the results have been obtained with the help of GAP.

This is joint work with Joanne Redden of Elmira College.

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