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Representation of Matroids by Homotopy Spheres

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The Topological Representation Theorem for oriented matroids, loosely put, says that any oriented matroid is topologically “close” to being realizable as an arrangement of hyperplanes in R^n. More specifically, any oriented matroid can be represented by an arrangement of topological equators on a sphere.

There is no reason to expect a similar result for non-orientable matroids. For instance, if a matroid M arises from an arrangement of hyperplanes in F^n, where F is a finite field, why should there be anything like an arrangement of equators on the sphere representing M? However, several years ago Ed Swartz proved a truly weird “Topological Representation Theorem for Matroids”, representing any matroid by an arrangement of homotopy spheres. His representations are far from canonical, and his construction is not explicit. I will describe a new construction whose end result is slightly weaker than Swartz's but which is (almost) canonical, completely explicit, and indeed quite simple.
