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Counting Permutations by Congruence Class of Major Index

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Let S_n be the symmetric group of all permutations of {1,2,…,n}. A permutation $\pi = a_1\ a_2\ \ldots\ a_n$ in S_n (written in one-line form) has major index

maj π = Sum$_{a\text{_{}i}$ > a_{i+1}}   i,

i.e., maj π is the sum of all the indices i where π has a descent. The major index is an important statistic in combinatorics and has many interesting properties.

Now fix two positive integers k, l which are relatively prime (i.e., have no common factors) and are at most n. Let $m_n^{k,l}$ be the k×l matrix whose (i,j) entry is the cardinality of the set

{π in S_n : maj π = i (mod k) and maj π^{-1} = j (mod l)}.

Surprisingly, this matrix has all its entries equal! We will outline a combinatorial proof of this theorem and other related results.

This is joint work with Helene Barcelo and Sheila Sundaram.
