Abstract for the Combinatorics Seminar 2005 November 15

Angle sums on polytopes have been studied in lower dimensions since classical geometry, and in more generality for about a century. First, we associate an angle to each face of an embedded polytope, where the angle is defined by considering the fraction of directions from the face which go into the interior of the polytope. An alternate way of thinking about this is to center an arbitrarily small full- dimensional sphere at an interior face of this point and take the volume of the sphere inside the polytope divided by the total volume of the sphere to get the angle. We get the i-th angle sum by adding the angles at all i-dimensional faces. Gram's relation says that the alternating sum of i-th angle sums of a polytope from i = 0 to d is 0. When d = 2, this reduces to the formula for the sum of angles of a polygon. Other relations are known for special classes of polytopes.

More recently, angle sums have been considered on complexes made from polytopes and similar results have been found to hold.

First I will survey the relations on angle sums of polytopes and make some connections to relations on face numbers. Then I will show how these results can be extended to polytopal complexes. This gives the relations on angle sums for a variety of complexes, including some triangulations of surfaces of genus g.
