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GCD Determinants

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In 1876, H. J. S. Smith proved the following beautiful determinantal identity. Let M be an $n \times n$ matrix with entries $m_{i,j}=\gcd(i,j)$, where, as usual, gcd stands for the greatest common divisor. Then

$\det M = \varphi(1)\, \varphi(2) \cdots \varphi(n)$,

where φ is the Euler phi-function, i.e., φ(n) is the number of positive integers m less than or equal to n with gcd(m,n)=1. Since Smith's paper, a host of generalizations and analogues have appeared in the literature. I will show that many of them are special cases of a simple identity in the incidence algebra of an arbitrary poset P. Smith's original result then follows by Möbius inversion when P is the lattice of divisors of n.

This is joint work with E. Altinisik and N. Tuglu.