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### GCD Determinants

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#### Abstract for the Combinatorics Seminar 2005 April 22

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In 1876, H. J. S. Smith proved the following beautiful determinantal identity. Let  $M$  be an  $n \times n$  matrix with entries  $m_{i,j} = \gcd(i,j)$ , where, as usual,  $\gcd$  stands for the greatest common divisor. Then

$$\det M = \varphi(1) \varphi(2) \cdots \varphi(n),$$

where  $\varphi$  is the Euler phi-function, i.e.,  $\varphi(n)$  is the number of positive integers  $m$  less than or equal to  $n$  with  $\gcd(m,n)=1$ . Since Smith's paper, a host of generalizations and analogues have appeared in the literature. I will show that many of them are special cases of a simple identity in the incidence algebra of an arbitrary poset  $P$ . Smith's original result then follows by Möbius inversion when  $P$  is the lattice of divisors of  $n$ .

This is joint work with E. Altinisik and N. Tuglu.

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