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Dowling Lattices, Quasigroups, and Latin Squares

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A quasigroup is a group without associativity or identity: what remains is unique solvability of the equation $xy=z$ given any two of x , y , and z . Its multiplication table is a Latin square, and conversely; so if we arbitrarily rearrange the rows and columns and relabel the entries of the Latin square we always get another quasigroup, equivalent to the first one. Not all quasigroups are equivalent to groups: just those that are associative.

The Dowling lattices of a group are geometries of arbitrary rank (or dimension) that geometrically encode the group structure. Dowling lattices of quasigroups also exist—and they encode equivalence classes of quasigroups—but only in rank 3 (dimension 2). The reason is that higher rank implies the associative law.

Dowling lattices can be described graph-theoretically as "biased expansions" of complete graphs. (I will define this.) Biased expansions of arbitrary graphs therefore generalize Dowling lattices and quasigroups. Some biased expansion graphs come from groups, but I will show how to construct many that do not. A generalized quasigroup operation is represented as a circle in the biased expansion graph and an associative property as chords of that circle; if there are few enough chords (in a certain sense), then associativity may fail and the generalized quasigroup does not arise from a group.

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