Matthias Beck (Binghamton)

The Linear Diophantine Problem of Frobenius

Abstract for the Combinatorics and Number Theory Seminar 2002 February 5

Given relatively prime positive integers $a_1, \ldots, a_n$, we call an integer t *representable* if there exist nonnegative integers $m_1, \ldots, m_n$ such that

$t = m_1 a_1 + \ldots + m_n a_n$.
We study the *linear diophantine problem of Frobenius*: namely, to find the largest integer which is not representable.

We translate this problem into a geometric one: consider N(t), the number of nonnegative integer solutions ($m_1, \ldots, m_n$) to $m_1 a_1 + \ldots + m_n a_n = t$ for any positive integer t. N(t) enumerates the integer points in a polytope. Solving the Frobenius problem now simply means finding the largest zero of N(t). N(t) turns out to be a quasi-polynomial, thereby yielding a straightforward analytic tool to recover and extend some well-known results on this classical problem.