Joseph P.S. Kung

Goncharov Polynomials and Parking Functions

Abstract for the Combinatorics and Number Theory Seminar 2001 December 14

Let **u** be a sequence of non-decreasing positive integers. A **u**-parking function of length n is a sequence $(x_1,x_2,\dots,x_n)$ whose order statistics (the sequence $(x_{(1)},x_{(2)},\dots,x_{(n)})$ obtained by rearranging the original sequence in non-decreasing order) satisfy $x_{(i)} \Leftarrow u_i$. The Goncharov polynomials $g_n(x; a_0,a_1,\dots,a_{n-1})$ are polynomials defined by the biorthogonality relation:

epsilon(a_i) D^i $g_n(x; a_0,a_1,\dots,a_{n-1})$ = n! delta_{in} ,

where epsilon(a) is evaluation at a. Goncharov polynomials form a ``natural basis of polynomials for working with **u**-parking functions. For example, the number of **u**-parking functions of length n is $(-1)^n g_n(0; u_1,u_2, \dots,u_n)$. Goncharov polynomials also satisfy a linear recursion obtained by expanding x^n as a linear combination of Goncharov polynomials. The combinatorial structure underlying this recursion is a decomposition of an arbitrary sequence of positive integers into two subsequences: a ``maximum **u**-parking function and a subsequence consisting of terms of higher values. From this combinatorial decomposition, we derive linear recursions for sum enumerators, expected sums of **u**-parking functions, and higher moments of sums of **u**-parking functions. These recursions yield explicit formulas for these quantities in terms of Goncharov polynomials.

This is joint work with Catherine Yan.