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Goncharov Polynomials and Parking Functions

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Let \mathbf{u} be a sequence of non-decreasing positive integers. A \mathbf{u} -parking function of length \mathbf{n} is a sequence $(x_1, x_2, ..., x_n)$ whose order statistics (the sequence $(x_{(1)}, x_{(2)}, ..., x_{(n)})$) obtained by rearranging the original sequence in non-decreasing order) satisfy $\mathbf{x}_{(i)} \leftarrow \mathbf{u}_i$. The Goncharov polynomials $\mathbf{g}_{\mathbf{n}}(\mathbf{x}; \mathbf{a}_0, \mathbf{a}_1, ..., \mathbf{a}_{n-1})$ are polynomials defined by the biorthogonality relation:

epsilon(a_i) D^{i} $g_{n}(x; a_{0}, a_{1}, ..., a_{n-1}) = n!$ delta_{in},

where epsilon(a) is evaluation at a. Goncharov polynomials form a ``natural basis of polynomials for working with \mathbf{u} -parking functions. For example, the number of \mathbf{u} -parking functions of length \mathbf{n} is $(-1)^n$ $\mathbf{g}_n(0; \mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_n)$. Goncharov polynomials also satisfy a linear recursion obtained by expanding \mathbf{x}^n as a linear combination of Goncharov polynomials. The combinatorial structure underlying this recursion is a decomposition of an arbitrary sequence of positive integers into two subsequences: a ``maximum \mathbf{u} -parking function and a subsequence consisting of terms of higher values. From this combinatorial decomposition, we derive linear recursions for sum enumerators, expected sums of \mathbf{u} -parking functions, and higher moments of sums of \mathbf{u} -parking functions. These recursions yield explicit formulas for these quantities in terms of Goncharov polynomials.

This is joint work with Catherine Yan.

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