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Biased Graphs and the Associative Law

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A quasigroup is like a group but without the identity, inverses, or associativity; all that is left is the multiplication table, which is an arbitrary Latin square. This is worth something: one has unique solvability of equations xy=z. Also, an identity can always be found. The crucial missing property is the associative law.

A *biased graph* is a graph together with a distinguished class of circles (a.k.a. circuits, cycles, polygons) that satisfies a certain combinatorial property. Each quasigroup with m elements gives rise to a kind of biased graph called an m-fold biased expansion of K_3, the complete graph of order 3. Conversely, every m-fold biased expansion of K_3 is obtained from a quasigroup. Trying to generalize this construction of quasigroups to K_n fails to be interesting because K_4 implies the associative law, so that a biased expansion of K_n must come from a group if $n > 3$.

We explore this fact and possible ways of getting around it. For instance, an m-fold biased expansion of C_n , the circle of length n, encodes an n-dimensional Latin hypercube, which might be considered the multiplication table of an (n-1)-ary operation. Chords in the circle imply specific associative properties which get stronger and stronger until when all chords have been added (so the graph becomes K_n) there is complete associativity, making the operation a group operation. What happens in between is completely unknown.