Matthias Beck

Counting lattice points by means of the residue theorem

Abstract for the Combinatorics and Algebra Seminars 1999 February 2 and 9

We use residues to count lattice points. More precisely, we use the residue theorem to compute the number of lattice points in a dilated n-dimensional tetrahedron with vertices at lattice points on each coordinate axis and the origin, known as the Ehrhart polynomial. We prove the Ehrhart-Macdonald reciprocity law for these tetrahedra relating the Ehrhart polynomials of the interior and the closure of the tetrahedra. To illustrate our method, we compute the Ehrhart coefficients for codimension 2 and 3. We show how our ideas can be used to compute the Ehrhart polynomial for an arbitrary convex lattice polytope, using residues of integrals in several variables. Finally, we apply our method to rational polytopes, i.e., polytopes with rational vertices. As an application to the rational case, we discuss Frobenius's coin exchange problem.