Spring 2023

- **January 24**
  Speaker: N/A
  Title: Organizational Meeting
  Abstract: We will discuss plans for this semester

- **January 31**
  Speaker: Alexander Borisov (Binghamton)
  Title: The abc-polynomials
  Abstract: To every triple of coprime natural numbers a=b+c, one can associate an interesting polynomial $f_{abc}(x)=\frac{bx^a-ax^b+c}{(x-1)^2}$. These abc-polynomials were introduced and studied in my old (1998) paper: http://people.math.binghamton.edu/borisov/documents/papers/abc.pdf In that paper I proved several results on the irreducibility of these polynomials, with the main theorem being that almost all of them, in the density sense, are irreducible. These results can certainly be strengthened and generalized in a number of ways, and the primary reason why this hasn't been done is that, probably, few people tried. I will give an overview of the motivation, results, and methods of this paper, in the hope that some of the graduate students in the audience get interested in pushing this topic further.

- **February 7**
  Speaker: Sailun Zhan (Binghamton)
  Title: Punctual Hilbert schemes of points of affine spaces in the Grothendieck group of varieties
  Abstract: We will give an introduction to punctual Hilbert schemes of n points of affine spaces A^{m+1}, which parametrizes (x_0,…,x_m)-primary ideals in k[x_0,…,x_m] of codimension n. Then we will give an explicit stratification of it with respect to m-dimensional partitions in the Grothendieck group of varieties. We will illustrate the idea by the cases when m=1 and 2.

- **February 14**
  Speaker: Sailun Zhan (Binghamton)
  Title: Punctual Hilbert schemes of points of affine spaces in the Grothendieck group of varieties, part 2
  Abstract: This is a continuation of the February 7 talk.

- **February 21**
  Speaker: Sayak Sengupta (Binghamton)
  Title: Heights in Diophantine Geometry
  Abstract: The height of a rational number is a measure of its arithmetic complexity. For example, although the numbers 5 and 500000/100001 are close, the second is arithmetically more complex, and has a much larger height. The height of a rational number is easy to define: it is simply the maximum of the absolute value of the numerator and the denominator when the number is expressed in lowest form. It is not immediately clear how one can extend this definition to more general algebraic numbers, such as the square root of 2. In this talk we will start with the definition of height of points in $\mathbb{Q}$ and build our way up to the general definition of heights of points in $\mathbb{P}^n_K$, K being a number field.

- **February 28**
  Speaker: Alexander Borisov (Binghamton)
  Title: Some applications of the Lagrange interpolation formula
  Abstract: This elementary talk will highlight two applications of the Lagrange interpolation formula. The first one is my old result on the “split Rolle closure” of the rational numbers

http://people.math.binghamton.edu/borisov/documents/papers/cb2.pdf and the second one is my solution of a conjecture by Dinesh Thakur, see Theorem 6 in https://arxiv.org/abs/2211.01076

- **March 7**
  Speaker: Alexander Borisov (Binghamton)
  Title: Discrete invariants of compactifications of the affine plane
  Abstract If a complete algebraic surface contains an affine plane, its discrete invariants are determined by the weighted graph (tree) of the curves at infinity. I will describe this structure and some associated invariants. The talk will be based on my 2014 paper http://people.math.binghamton.edu/borisov/documents/papers/divisorialvaluations.pdf I will also describe some recent improvements by my student Patrick Carney.

- **March 14**
  Speaker: Mithun Padinhare Veettil (Binghamton)
  Title: Nilpotent and Geometrically Nilpotent Subvarieties.
  Abstract: In this talk, we will define nilpotent and geometrically nilpotent subvarieties, an idea grew that out of density theorem proved by Prof. Borisov. Then we will discuss two explicit constructions that give rise to geometrically nilpotent but not nilpotent subvarieties (in fact, infinitely many of them!)

- **March 21**
  Speaker: Alexander Borisov (Binghamton)
  Title: Discrete invariants of compactifications of the affine plane , part 2
  Abstract: This is the continuation of the March 7 talk. I plan describe the determinant labels and Patrick Carney's improvements to the way they can be calculated

- **March 28**
  Speaker: Sayak Sengupta (Binghamton)
  Title: Elliptic curves and heights in Elliptic curves
  Abstract: Elliptic curve has been a central object of study in arithmetic geometry and number theory for a long time. In this talk we will explore the richness of elliptic curves as an abelian group with more emphasis on examples rather than proofs of theorems. We will define the two height concepts, viz. naive and canonical heights, on elliptic curve $E/\mathbb{Q}$ and build our way up to defining the well-known N\'eron-Tate pairing attached to E which is a non-degenerate, symmetric, bilinear form on $E(\mathbb{Q})$, modulo the torsion points.

- **April 1-2**
  Special Event: Eleventh Upstate New York Number Theory Conference, at the University of Rochester https://sites.google.com/view/upstate-ny-nt-conf-2023/home

- **April 11** (by Zoom: Zoom link)
  Speaker: John Lee (UIC)
  Title: Acyclic Reduction of Elliptic Curves for Primes in Arithmetic Progression
  Abstract: Let E be an elliptic curve defined over $\mathbb{Q}$ and p a rational prime. $\tilde{E}_p$ denotes the reduction of E modulo p. Recently, Akbal and G \"{u}lo\v{g}lu considered the question of cyclicity of $\tilde{E}_p(\mathbb{F}_p)$ under the restriction that lies in an arithmetic progression. In this talk, we study the issue of which arithmetic progressions k mod n have the property that, for all but finitely many primes $p \equiv k$ mod n, the group $\tilde{E}_p(\mathbb{F}_p)$ is NOT cyclic. Furthermore, we study the statistical congruence class bias of primes of cyclic reduction for generic elliptic curves. This is a joint work with

Nathan Jones.

- **April 18** (by Zoom: Zoom link)
  Speaker: Paolo Dolce (Ben-Gurion University)
  Title: Introduction to Diophantine approximation and a generalization of Roth’s theorem
  Abstract: Classically, Diophantine approximation deals with the problem of studying “good” approximations of a real number by rational numbers. I will explain the meaning of “good approximants” and the classical main results in this area of research. In particular, Klaus Roth was awarded with the Fields medal in 1955 for proving that the approximation exponent of a real algebraic number is 2. I will present a recent extension of Roth’s theorem in the framework of adelic curves. These mathematical objects, introduced by Chen and Moriwaki in 2020, stand as a generalization of global fields.

- **April 25**
  Speaker: Hari Asokan (Binghamton)
  Title: Geometric invariant theory
  Abstract: Geometric invariant theory is a method for constructing quotients by group actions in algebraic geometry, developed by David Mumford in 1965. GIT theory is used to construct moduli spaces of geometric objects in algebraic geometry. The theory also has interactions with differential geometry and symplectic geometry.
  In this talk we will discuss some basic concepts and definitions in geometric invariant theory. We will focus on the congruence action of special linear group on multiple quadratic forms. The geometric invariant theory of multiple quadratic forms is motivated by applications to the study of quadratic forms and, due to the recent work of Buium-Vasiu, of \delta-modular forms on moduli spaces of principally polarized abelian varieties of arbitrary dimension.

- **May 1** (Monday), 3:30-5:30 pm Special Event: PhD Defense
  Speaker: Patrick Carney (Binghamton)
  Title: Application of the Theory of Farey Fractions to the Combinatorics of Some Compactifications of the Affine Plane
  Abstract: The combinatorial structure of the algebraic surfaces obtained from the complex projective plane by a sequence of blow-ups of points at infinity is classically described by a certain weighted graph whose vertices correspond to irreducible curves at infinity. In 2014, Alexander Borisov associated to each such curve a pair of integers that are invariant under subsequent blow-ups. We investigate these invariants and furnish a much more efficient method of calculating them based on the classical theory of Farey fractions.