Spring 2021

- **February 16**
  Speaker: N/A
  Title: Organizational Meeting
  Abstract: We will discuss plans for this semester

- **February 23**
  Speaker: Fikreab Solomon Admasu (Binghamton)
  Title: Bhargava's composition law for binary cubic forms
  Abstract: The Delone-Fadeev correspondence shows that binary cubic forms with integer coefficients parametrize orders in cubic fields. With this result in mind, Bhargava constructs a binary cubic form from 2x3x3 boxes of integers and proves that there is a natural composition law for the boxes of integers. The group resulting from this law is then shown to be isomorphic to the class group of a corresponding cubic order. This is a cubic analogue of Gauss's theory of composition for binary quadratic forms and its relation to ideal classes of quadratic orders. The talk is based on Bhargava's “Higher composition laws II: On cubic analogues of Gauss composition.”

- **March 2**
  Speaker: Hyuk Jun Kweon (MIT)
  Title: Bounds on the Torsion Subgroups of Néron-Severi Group Schemes
  Abstract: Let $X \hookrightarrow \mathbb{P}^r$ be a smooth projective variety defined by homogeneous polynomials of degree $\leq d$ over an algebraically closed field k. Let $\mathbf{Pic}\, X$ be the Picard scheme of X, and $\mathbf{Pic}\, ^0 X$ be the identity component of $\mathbf{Pic}\, X$. The N\'eron–Severi group scheme of X is defined by $\mathbf{NS} X = (\mathbf{Pic}\, X)/(\mathbf{Pic}\, ^0 X)_{\mathrm{red}}$, and the N\'eron–Severi group of X is defined by $\mathrm{NS}\, X = (\mathbf{NS} X)(k)$. We give an explicit upper bound on the order of the finite group $(\mathrm{NS}\, X)_{{\mathrm{tor}}}$ and the finite group scheme $(\mathbf{NS} X)_{{\mathrm{tor}}}$ in terms of d and r. As a corollary, we give an upper bound on the order of the torsion subgroup of second cohomology groups of X and the finite group $\pi^1_{et}(X,x_0)^{\mathrm{ab}}_{\mathrm{tor}}$. We also show that $(\mathrm{NS}\, X)_{\mathrm{tor}}$ is generated by $(\deg X -1)(\deg X - 2)$ elements.

- **March 9**
  Speaker: Thomas Morrill (Trine)
  Title: Sign Changes in the Prime Number Theorem
  Abstract: The prime number theorem is a lovely part of 19th century mathematics, leveraging the techniques of complex analysis in order to find results in the discrete setting, namely that the number of primes between 1 and x may be estimated by $x/log(x).$ However, to an analyst, the theorem gives a statement about the asymptotic growth of two real-valued functions. In fact, the recognizable $\pi(x) \sim x/log(x)$ is one of several equivalent statements of the theorem, depending on how one chooses to weight the count of primes from 1 to x. We cover the fundamentals of this topic, and present our results on the sign of $\phi(x) - x$ as $x\to \infty$.

- **March 16**
  Speaker: Fikreab Solomon Admasu (Binghamton)
  Title: Bhargava's composition law for binary cubic forms 2
  Abstract: This is a continuation of the talk from February 23 with a focus on illustrative examples and

some questions.

- **March 23**
  Speaker: Alexander Borisov (Binghamton)
  Title: Coherent sheaves over the compactified Spec(Z)
  Abstract: Lattices in finite-dimensional Euclidean spaces can be viewed as Arakelov analogs of locally free sheaves of finite rank over a compact curve. There are many indications that this category can be extended to a category of “coherent sheaves”. I will discuss some motivation, basic definitions, and and some work in progress, joint with Jaiung Jun.

- **March 30**
  Speaker: Andrew Lamoureux (Binghamton)
  Title: Differential Algebra
  Abstract: This talk will explore the basics of differential algebra, such as differential rings, differential ideals, and the ring of differential polynomials. I will then focus on differential field theory, emphasizing its parallels to ordinary field theory.

- **April 6**
  Speaker: Biao Wang (SUNY Buffalo)
  Title: Alladi's formula and its analogues
  Abstract: In 1977, Alladi introduced a duality between prime factors of integers, from which along with the prime number theorem in arithmetic progressions he showed the following beautiful formula on the M\"{o}bius function $\mu(n)$: if $(\ell,k)=1$, then $$ -\sum_{\begin{smallmatrix}n\ge 2\\ p_{\min}(n)\equiv \ell (\operatorname{mod}k) \end{smallmatrix}}\frac{\mu(n)}{n}=\frac1{\varphi(k)},$$ where $p_{\min}(n)$ is the smallest prime factor of n and φ is the Euler's totient function. Recently, this formula has been generalized from the Chebotarev density to the natural density of sets of primes in works by Dawsey, Sweeting and Woo, Kural, McDonald and Sah. In this talk I will discuss the generalizations of these results to other arithmetic functions and the analogues over function fields.

- **April 12 (Monday), 12-1 pm**
  Speaker: James Myer (CUNY)
  Title: The alterations paradigm shift for the problem of resolution of singularities
  Abstract: Heisuke Hironaka solved the problem of resolution of singularities for a variety over a field of characteristic zero in 1964, although the story goes that his proof was so complicated as to stump Alexander Grothendieck! Efforts have been made since then to simplify the proof and see if it can be made to work for positive characteristic, but to no avail.
  Regardless of characteristic: The singularities of a curve are resolved in one fell swoop by the normalization, a powerful tool hailing from algebra made to work for us in geometry, by Jean-Pierre Serre’s Criterion for Normality, and we owe much to Oscar Zariski for teaching us about the normalization. Joseph Lipman dealt with surfaces in as much generality as one could hope for. Threefolds are rumored to have been handled by Vincent Cossart and Olivier Piltant, although their proof is quite long and there are some restrictions on the characteristic. Fourfolds and up are uncharted territory for the most part.
  In his 1996 paper Smoothness, Semi-Stability, and Alterations, Johan de Jong introduces a paradigm shift for solving the problem of resolution of singularities by relaxing a resolution of singularities to what he calls an alteration. The distinction is that a resolution is a birational morphism (generically one-to-one), whereas an alteration is a generically finite morphism (generically finite-to-one). Every variety can be altered to a nonsingular variety regardless of the characteristic. In fact, de Jong’s technique paired

up with Dan Abramovich's geometric insight yields a proof that every variety over a field of characteristic zero admits a resolution of its singularities in a paper consisting of only twelve pages, see Smoothness, Semi-Stability, and Toroidal Geometry. de Jong's technique relies on (at least) two ingenious ideas: The first is the statement that there is a simple blowup of any variety that admits a morphism to a projective space of one less dimension whose fibers are curves ~ intuitively, any variety can be modified to be a family of curves. The second is that the curves in the family of curves that results can be marked so as to become stable (in the sense of Pierre Deligne and David Mumford) so that it gives rise to morphism into the moduli space of stable curves, where we may take advantage of established facts about the moduli space of stable curves, including the fact that its compactification has curves with at worst nodal singularities. My talk has the goal of introducing Johan de Jong's paradigm shift, and indicating, to the extent that I can, the idea of the proof that every variety may be altered to a nonsingular variety. An emphasis will be placed on the picture that demonstrates that there is a simple blowup of any variety that admits a morphism to a projective space of one less dimension whose fibers are curves.

- **April 27**
  Speaker: Andrew Lamoureux (Binghamton)
  Title: Differential Algebra (continued)
  Abstract: This is a continuation of my talk from March 30. In particular, I will discuss the basics of differential Galois theory and then p-derivations, which are arithmetic analogues of derivations.

- **May 4**
  Speaker: Francois Greer (IAS)
  Title: Cycle-valued quasi-modular forms
  Abstract: The theta correspondence of automorphic forms produces a rich supply of modular forms with "special cycle" coefficients in the cohomology of Shimura varieties. I will describe what happens when we take the closure of the special cycles in toroidal compactifications, focusing on the case of orthogonal type. This has concrete applications in enumerative algebraic geometry, and leads to a related program outside the Shimura variety context.

- **May 11**
  Speaker: Garen Chiloyan (UConn)
  Title: A study of isogeny-torsion graphs
  Abstract: An isogeny-torsion graph is a nice visualization of the $\mathbb{Q}$-isogeny class of an elliptic curve defined over $\mathbb{Q}$. A theorem of Kenku shows sharp bounds on the number of distinct isogenies that a rational elliptic curve can have (in particular, every isogeny graph has at most 8 vertices). In this talk, we classify what torsion subgroups over $\mathbb{Q}$ can occur at each vertex of a given isogeny-torsion graph of elliptic curves defined over the rationals. Then we will determine which isogeny-torsion graphs correspond to an infinite set of $\textit{j}$-invariants. This is joint work with \'Alvaro Lozano-Robledo.