# Fall 2024

- **August 27**
  ***Speaker***: N/A
  ***Title***: Organizational Meeting
  ***Abstract***: We will discuss plans for this semester

- **September 3** (Algebra Seminar, cross-listed, **2:50-3:50 pm**)
  ***Speaker***: Dikran Karagueuzian (Binghamton)
  ***Title***: Elliptic Curves for Dummies
  ***Abstract***: A theorem on the random variable of inverse image sizes for a polynomial over a finite field of order q computes the moments of the random variable of inverse images sizes up to an error term. This error term decreases with the inverse of the square root of q. Standard results in the theory of elliptic curves will be used to show that this error term cannot generally be improved. No familiarity with the extensive theory of elliptic curves will be assumed.

- **September 10**
  ***Speaker***: Alexander Borisov (Binghamton)
  ***Title***: A Primer on Modular Forms
  ***Abstract***: I will give a very brief introduction to the theory of modular forms. The topics will include the action of the group of invertible 2×2 integer matrices on the upper half plane, the classical fundamental region, the modular functions and forms of even weight, the Eisenstein series. If time permits, I will also talk about Hecke operators, Petersson inner product, and forms of higher level. The main reference for the talk is the last chapter of the classical book by J.-P. Serre “A Course in Arithmetic”.

- **September 17** (Unusual time: **4:45-5:45 pm**)
  ***Speaker***: Noah Stephens-Davidowitz (Cornell)
  ***Title***: A reverse Minkowski theorem
  ***Abstract***: Minkowski's celebrated first theorem is one of the foundational results in the study of the geometry of numbers, and it has innumerable applications from number theory to convex geometry to cryptography. It tells us that a lattice (i.e., a linear transform of Z^n) that is globally dense (i.e., has low determinant) must be locally dense (i.e., must have many short vectors). We will show a proof of a nearly tight converse to Minkowski's theorem, originally conjectured by Daniel Dadush—a lattice with many short points must have a sublattice with small determinant. This “reverse Minkowski theorem” has numerous applications in, e.g., complexity theory, additive combinatorics, cryptography, the study of Brownian motion on flat tori, algorithms for lattice problems, etc. Just recently, it was used by Reis and Rothvoss to give the first asymptotic improvement to integer programming in nearly forty years.
  Based on joint work with Oded Regev.

- **September 24**
  ***Speaker***: Alexander Borisov (Binghamton)
  ***Title***: Locally Integer Polynomial Functions on Infinite Subsets of Integers
  ***Abstract***: A locally integer polynomial function on a subset of $\mathbb Z$ is an integer-valued function whose restriction to any finite subset is given by a polynomial with integer coefficients. For infinite domains these functions have some properties reminiscent of the properties of complex analytic functions. For example, a LIP function that takes value 0 at infinitely many inputs must be zero, so a “LIP continuation” from a smaller infinite set to a larger one is unique, if exists. The talk will be partially based on the preprint

https://arxiv.org/pdf/2401.17955 but will also include more recent results related to LIP continuation and the structure of the corresponding rings. I spoke about this topic in the beginning of last semester, but will not assume any knowledge about the subject. A number of open questions will be proposed.

- **October 15**
  ***Speaker***: Hari Asokan (Binghamton)
  ***Title***: GIT quotient of SL_g action on symmetric matrices
  ***Abstract***: The special linear group SL_g acts on the vector space of symmetric matrices, V_g by congruence action. This action extends to (V_g)^{r+1}. We are interested in the GIT quotient and the invariant ring of this action. In this talk we will discuss a linear algebra problem arising from the above situation.

- **October 22**
  ***Speaker***: Anton Mosunov (Cornell)
  ***Title***: On the Area of the Fundamental Region of a Binary Form
  ***Abstract***: Let F(x,y) be a binary form with integer coefficients, of degree n > 2 and nonzero discriminant D_F. Let A_F denote the area of the fundamental domain {(x,y) in R^2 : |F(x,y)|\leq 1}. Back in the 90s my academic brother, Michael Bean, proved that the quantity |D_F|^{1/n(n-1)}*A_F achieves its maximum over all forms specified above when F(x,y) = xy(x-y). Moreover, when n is fixed, he conjectured that the maximum must be attained by the form
  F_n*(x,y) = \prod_{k=1}^n (sin(kπ/n)*x – cos(kπ/n)*y)
  I will talk about the recent work on this conjecture, as well as about a similar problem which concerns bounding the quantity h_F^{2/n} *A_F from below. Here h_F is an appropriately chosen height of F.

- **October 29**
  ***Speaker***: Jaiung Jun (SUNY New Paltz, IAS)
  ***Title***: Schemes over the natural numbers
  ***Abstract***: In this talk, I will first explain how a notion of positivity in algebraic geometry / number theory could be captured in terms of semirings by providing an example of the narrow class group of a number field as a reflexive Picard group. Then, I will introduce a notion of equivariant vector bundles over the natural numbers, and prove a version of Klyachko classification theorem of toric vector bundles in this setting.

- **November 5**
  ***Speaker***: Mithun Veettil (Binghamton)
  ***Title***: Kirch and Golomb Topology on $\mathbb N$ and Locally Integer Polynomials
  ***Abstract***: Golomb topology on a domain is a generalization of arithmetic topology on $\mathbb N$. In this talk, we will show that Golomb topology is connected on $\mathbb N$ but not locally connected. But, by modifying the basis ”a little bit” we obtain a new topology on $\mathbb N$, known as Kirch topology, and we will prove that $\mathbb N$ is locally connected with this new topology. With these topologies, we will explore the structure of locally integer polynomials on $\mathbb N$.

- **November 12**
  ***Speaker***: Chris Schroeder (Binghamton)
  ***Title***: The O'Nan Scott Theorem
  ***Abstract***: The O'Nan Scott Theorem classifies the maximal subgroups of the finite symmetric groups. It is arguably the most influential theorem in the theory of permutation groups, and it has far-reaching consequences in finite group theory in light of the classification of finite simple groups. In this talk, which is a continuation of the talk in the Algebra Seminar immediately beforehand, we will prove it.

- **December 3**
  ***Speaker***: Marcin Mazur (Binghamton)
  ***Title***: Zsigmondy's Theorem
  ***Abstract***: In my earlier talk in the Algebra Seminar I showed how Zsigmondy's theorem can be used to get simple proofs of some fundamental results in algebra. In this expository lecture I will provide an elementary proof of Zsigmondy's theorem which will showcase some nice techniques from elementary number theory.