The Algebra Seminar

Unless stated otherwise, the seminar meets Tuesdays in room WH-100E at 2:50 p.m. There will be refreshments served at 4:00 in room WH-102.

Organizers: Alex Feingold and Marcin Mazur

To receive announcements of seminar talks by email, please join the seminar's mailing list.

Spring 2018

- **January 16**
  Jonas Deré (KU Leuven Kulak)
  Which manifolds admit expanding maps

  Abstract: In 1981, M. Gromov completed the proof that every manifold admitting an expanding map is, up to finite cover, homeomorphic to a nilmanifold. Since then it was an open question to give an algebraic characterization of the nilmanifolds admitting an expanding map. During my talk, I will start by introducing the basic notions of expanding maps and nilmanifolds. Then I explain how the existence of such an expanding map only depends on the covering Lie group and on the existence of certain gradings on the corresponding Lie algebra. One of the applications is the construction of a nilmanifold admitting an Anosov diffeomorphism but no expanding map, which is the first example of this type.

- **January 23**
  Organizational Meeting
  Title of Talk

  Abstract: Please come or contact the organizers if you are interested in giving a talk this semester or want to invite someone.

- **January 30**
  Casey Donoven (Binghamton University)
  Subgroups and Supergroups of Thompson's Group V

  Abstract: Thompson's group V is a group of homeomorphisms of Cantor space. It acts by finite prefix exchanges

on infinite sequences over a finite alphabet. These finite prefix exchanges can be restricted or augmented to form subgroups and supergroups of V. I will describe these groups and when they are isomorphic (conjugate) to V.

- **February 6**
  Jaiung Jun (Binghamton University)
  Algebraic geometry in characteristic one

  Abstract: Algebraic geometry in characteristic one (also known as Geometry over the ``field with one element'') is a recent area of mathematics emerging from certain heuristic ideas relating combinatorics, algebraic geometry, and number theory. Two main driving forces of the theory are to understand combinatorial objects in an algebraic geometric way and to translate the proof of Weil conjectures to the case of Riemann hypothesis. In this talk, we outline the meaning of working in characteristic one and highlight two particular perspectives of the theory (tropical geometry and algebraic geometry over hyperrings).

- **February 13**
  Ben Brewster (Binghamton University)
  Sylow Intersections and the Chermak-Delgado Lattice

  Abstract: In a first graduate course focussed on Group Theory, being able to count non- identity elements when distinct Sylow subgroups intersect trivially, is a memorable experi- ence. There are numerous etensions and intensive examination of these Sylow Intersections. One of these is known as Brodkey's Theorem. It is closely tied to the existence of a regular orbit under a suitable group action. In the seminar, I will give full definitions and exhibit some of the points mentioned above. Recently R. McCulloch and M. Tarnauceanu have, using suggestions from I.M. Isaacs, been able to find application of these ideas to the Chermak-Delgado Lattice of a finite group. Recall if G is finite and $H \leq G$, one defines $m_G(H) = |H| |C_G (H)|$, $m^{*}(G) = \max\{m_G (H): H\leq G\}$, and $CD(G) = \{H: m_G (H) = m^{*}(G)\}$. It is known that $CD(G)$ is a modular self- dual sublattice of subgroup lattice of G. McCulloch and Tarnauceanu seek a classification of groups where $|CD(G)| = 1$. Ideas from above can be organized to prove a nice result. Theorem (Isaacs, McCulloch, Tarnauceanu 2017). If $G = AB$ where A and B are abelian, A is normal in G, and $(|A|, |B|) = 1$, then $CD(G) = \{AC_B(A)\}$.

- **February 20**
  Victor Protsak (Cornell University)
  Invariants of representations

  Abstract: One of the most fundamental questions in any field of mathematics is the Classification Problem: Determine all objects of a given kind up to some natural notion of equivalence. Next, comes the Recognition Problem: Given an object, identify its place in the classification. A general approach to these problems is via invariants, entities which are preserved under equivalences and can be explicitly or effectively computed. In Lie representation theory, we are interested in the classification of irreducible admissible representations of reductive Lie groups and of certain classes of modules over Lie algebras. Familiar invariants include the infinitesimal character, K-types, highest weight and the Weyl character. I will focus on invariants related to ideals in enveloping algebras, especially rank, the minimal polynomial and quantized elementary divisors. This approach has applications to description of certain noncommutative rings by generators and relations and to Howe duality.

- **February 27**
  Bruno Duchesne (L'Institut Élie Cartan de Lorraine)
  Kaleidoscopic groups

Abstract: Inspired by Burger-Mozes construction of universal groups acting a regular tree, we construct universal groups with a prescribed local action on dendrites. Dendrites are compacta that can be thought as continuous analogs of trees. The groups we construct are Polish groups with nice properties that reflect the higher transitivity properties of a given (infinite) permutation group. No knowledge of dendrites or Polish groups is required. This is joint work with Nicolas Monod and Phillip Wesolek.

- **March 6**
  No Classes
  Title of Talk

  Abstract: No meeting

- **March 13**
  Hung Tong-Viet (Binghamton University)
  Brauer characters and normal Sylow p-subgroups

  Abstract: The celebrated $It\hat{o}-Michler$ theorem for Brauer characters states that if a prime p does not divide the degrees of any irreducible p-Brauer characters of a finite group G, then G has a normal Sylow p-subgroup. In this talk, I will discuss several generalizations of this theorem using various inequalities involving p-parts and p'-parts of the p-Brauer character degrees.

- **March 13 (Cross Listing from Combinatorics Seminar. Time: 1:15-2:15. WH-100E)**
  Victor Reiner (Minnesota)
  Finite General Linear Groups and Symmetric Groups

  Abstract:In recent years we have seen surprising counting phenomena related to the symmetric group, with striking analogues for general linear groups over finite fields. Often the explanations come from invariant theory. I will give examples and pose some intriguing conjectures that come from pursuing the analogy further.

- **March 20**
  Joe Cyr (Binghamton University)
  Quandles and the "Abelian iff Quasi-Affine" Problem

  Abstract: The "abelian iff quasi-affine" problem in universal algebra asks what is the relationship between a generalized notion of abeliannes and the representability of an algebra by modules. I will introduce the concept of abeliannes for general algebras (which at first does not all seem to resemble the definition for groups) and present a recently published proof that indeed a quandle is abelian if and only it is quasi-affine. No prior knowledge about quandles or general algebras will be assumed.

- **March 27**
  Matt Evans (Binghamton University)
  Bounded commutative BCK-algebras do not form a discriminator variety

  Abstract: In this talk I will explain what it means for an algebra (in the sense of universal algebra) to be a discriminator algebra, and for a variety of algebras to be a discriminator variety. Discriminator varieties are well-studied and have many nice properties. For example, in a discriminator variety, the notions of simple, subdirectly irreducible, and directly indecomposable all coincide. I will show that the variety of bounded commutative BCK-algebras is not a discriminator variety.

- **March 27 (Cross Listing from Combinatorics Seminar. Time: 1:15-2:15. WH-100E)**
  Farbod Shokrieh (Cornell University)
  Effective Divisor Classes on Graphs

  Abstract: Graphs can be viewed as (non-Archimedean or tropical) analogues of Riemann surfaces. For example, there are well-behaved (and useful) notions of divisors, Riemann-Roch, and Abel-Jacobi theory on graphs. I introduce the notion of semibreak divisors, which provide nice representatives for effective divisor classes on graphs. I then discuss a few applications about the generic behavior of effective divisor classes, analogous to some classical results on Riemann surfaces. Proofs in this tropical setting are more subtle. This is joint work with Andreas Gross (Imperial College) and Lilla Tóthmérész (Cornell).

- **April 3**
  Spring Break
  Title of Talk

  Abstract: No meeting

- **April 10**
  Eric Bucher (Michigan State University)
  Introducing cluster algebras and their applications

  Abstract: Cluster algebras were first invented by Fomin and Zelevinsky in 2003 to study total positivity of canonical bases. Since their inception, these mathematical objects have popped up in a large variety of seemingly unrelated areas including: Teichmuller theory, Calabi-Yao categories, integrable systems, and the study of high energy particle physics. In this talk we will lay the basic groundwork for working with cluster algebras as well as discuss a few of their applications to the above areas. This talk is intended to be introductory so no background or definitions will be assumed. The intention is to have everyone walk away having learned about this new and fascinating algebraic object.

- **April 17**
  Rachel Skipper (Binghamton University)
  The congruence subgroup problem for a family of branch groups

  Abstract: A group, G, acting on a regular rooted tree has the congruence subgroup property (CSP) if every subgroup of finite index contains the stabilizer of a level of the tree. When the subgroup structure of G resembles that of the full automorphism group of the tree, additional tools are available for determining if G has the CSP. In this talk, we look at the Hanoi towers group which fails to have the CSP in a particular way. Then we will generalize this construction to a new family of groups and discuss the CSP for them. This talk is part of Rachel Skipper's doctoral dissertation defense. The examining committee consists of Benjamin Brewster, Matthew Brin, Patrick Madden, and Marcin Mazur (chair).

- **April 24**
  Gabriel Conant (Notre Dame University)
  VC-dimension, pseudofinite groups, and arithmetic regularity

  Abstract: I will discuss recent work on the structure of VC-sets in groups, i.e. subsets whose family of left translates has absolutely bounded VC-dimension. In joint work with A. Pillay, we show that VC-set A in a pseudofinite group G is “generically dominated” by a certain compact group of the form G/H, where H is a

canonical normal subgroup of G associated to A. Informally, this implies that almost all cosets of H are either almost contained in A or almost disjoint from A. More formally, if C is the set of cosets of H, which intersect both A and its complement in large sets with respect to the pseudofinite measure on G, then the Haar measure of C in G/H is zero. In joint work with A. Pillay and C. Terry, we use this generic domination to prove arithmetic regularity lemmas for VC-sets in finite groups. These results are motivated by Green's arithmetic regularity lemma for abelian groups, and generalize (without effective bounds), work of Alon-Fox-Zhao and Terry-Wolf on improved arithmetic regularity for VC-sets.

- **May 1**
  David Biddle (Binghamton University)
  Generating systems for finitely generated groups and their relation to special classes of characteristic subgroups of free groups

  Abstract: Neumann & Neumann introduced two strengthenings of the concept of characteristic subgroup in the early 1950's. In particular, given a group Γ and quotient group Q, they were interested in studying the set of all N so that $\Gamma/N\cong Q$. We show how these extended definitions of characteristic subgroups give rise to sequences of groups with extremely strong universal mapping properties with $\Gamma=F_n$ a free group and $Q=\cap_{\phi}\, \ker\phi$ where the intersection is over all members of the set $Epi(F_n,G)$ for a fixed group G

- Pre-2014 semesters
- Fall 2014
- Spring 2015
- Fall 2015
- Spring 2016
- Fall 2016
- Spring 2017
- Fall 2017
- Spring 2018