The Algebra Seminar

The seminar will meet in-person on Tuesdays in room WH-100E at 2:50 p.m. There should be refreshments served at 4:00 in room WH-102. As of Saturday, March 26, 2022, masks are optional.

Anyone wishing to give a talk in the Algebra Seminar this semester is requested to contact the organizers at least one week ahead of time, to provide a title and abstract. If a speaker prefers to give a zoom talk, the organizers will need to be notified at least one week ahead of time, and a link will be posted on this page.

If needed, the following link would be used for a zoom meeting (Meeting ID: 981 8719 2351) of the Algebra Seminar:

Algebra Seminar Zoom Meeting Link

Organizers: Alex Feingold, Daniel Studenmund and Hung Tong-Viet

To receive announcements of seminar talks by email, please join the seminar's mailing list.

Fall 2022

- **August 23**
  Organizational Meeting

  Please think about giving a talk in the Algebra Seminar, or inviting an outside speaker.

- **August 30**
  Alex Feingold (Binghamton University)
  Representations of Kac-Moody Lie Algebras

  Abstract: We will first review basic definitions and examples of finite dimensional Lie algebras and their representations. The infinite dimensional Heisenberg Lie algebra will be shown to have a representation on the space of polynomials in infinitely many variables. That space, V, is $\bf Z$-graded into subspaces V_n of total degree n with $\dim(V_n) = p(n)$, the classical partition function. Then we will give a brief introduction to the infinite dimensional Kac-Moody (KM) Lie algebras defined by generators and relations from a generalized Cartan matrix, A. We will discuss in detail the cases when A is 2×2, which give either affine or hyperbolic KM algebras. The affine case is related to the Heisenberg algebra, and we will present the root diagram and a weight diagram for an irreducible representation. We will discuss one hyperbolic example, Fib, and study its root system, and some irreducible representations (highest weight and non-standard).

- **September 6**
  No Algebra Seminar - Monday classes meet

- **September 13**
  Dikran Karagueuzian (Binghamton University)
  Moments of Polynomials over Finite Fields

  Abstract: A polynomial over a finite field can be regarded as a map of the finite field to itself. The variance of the inverse image sizes has been studied in connection with the question of whether such maps are a good substitute for random maps. We are able to show that polynomial maps are not random in the sense that all moments, not just the variance, must be integers, in an asymptotic sense as the size of the finite field becomes large. This is based on joint work with Per Kurlberg.

- **September 20**
  Daniel Studenmund (Binghamton University)
  Introduction to automorphism towers

  Abstract: The automorphism tower of a centerless group G is the increasing sequence $G < Aut(G) < Aut(Aut(G)) <$ etc., which is then extended to be ordinal-indexed at limit ordinals via unions. A group G is called complete if this sequence stabilizes at the second term — or, equivalently, if $Out(G)$ is trivial. Wielandt proved in the 1930s that this sequence stabilizes at some finite stage for any finite, centerless group G. Nearly 50 years later, Thomas proved that the sequence stabilizes without the “finite” hypothesis, at the expense of allowing the tower to stabilize at a possibly infinite ordinal. We will review basic definitions and constructions, and if time permits, compare in form to the Neukirch-Uchida theorem.

- **September 27**
  No Algebra Seminar - No classes

- **October 4**
  No Algebra Seminar - 1 PM Recess

- **October 11**
  Luke Elliot (Binghamton University)
  A connection between Thomson's Groups and (unindexed) two-sided subshifts of finite type

  Abstract: Two-sided subshifts of finite type are a class of topological dynamical system (or equivalently topological unary algebras) which have been studied for over 50 years. Most famously the problem of determining when two subshifts are isomorphic remains unsolved. Thompsons group V has been around for a similarly long amount of time and (together with T) was the first known example of a finitely presented infinite simple group. I plan to introduce both subshifts of finite type and V, and breifly discuss recently discoved connections between them.

- **October 18**

Nham Ngo (University of North Georgia - Gainesville)
Cohomology of SL_2

Abstract: Let k be an algebraically closed field of prime characteristic and SL_2 the rank 1 simple algebraic group defined over k. Cohomology of SL_2 is still unknown for many cases. In this talk, we will give a brief overview of representation and cohomology of this group. Some new calculations on bounding the dimension of SL_2-cohomology with coefficients in Weyl modules will be introduced. (This is joint work with Khang Pham.)

- **October 25**
  Hung Tong-Viet (Binghamton University)
  Variations of Baer-Suzuki theorem and applications

Abstract: The Baer-Suzuki theorem, a classical result in finite group theory, states that if C is a conjugacy class of a finite group G and if every two elements in C generate a nilpotent subgroup, then C generates a nilpotent normal subgroup of G. This gives a nice characterization of the Fitting subgroup of G, that is, the largest nilpotent normal subgroup of G. This theorem was originally proved by Baer and later by M. Suzuki in 1965. A more direct and elementary proof was obtained by Alperin and Lyons in 1971. Many generalizations of this theorem have been proposed and studied over the years. In this talk, I will discuss the proofs of this theorem and some of its variants and will give an application of these results to the character theory of finite groups.

- **November 1**
  Yash Madanha (University of Pretoria)
  Average number of zeros of characters of finite groups

Abstract: There has been some interest on how the average character degree affects the structure of a finite group. In this talk we will discuss an analogue of the average character degree: denoted by $ \mathrm{anz}(G) $, the average number of zeros of characters of a finite group $ G $ is defined as the number of zeros in the character table of $ G $ divided by the number of irreducible characters of $ G $. We show that if $ \mathrm{anz}(G) < 1 $, then the group $ G $ is solvable and also that if $ \mathrm{anz}(G) < \frac{1}{2} $, then $ G $ is supersolvable. We characterise abelian groups by showing that $ \mathrm{anz}(G) < \frac{1}{3} $ if and only if $ G $ is abelian. We shall also discuss some work by Moreto and some very recent work of Qian on this invariant.

- **November 8**
  Luise-Charlotte Kappe (Binghamton University, retired)
  A generalization of the Chermak-Delgado lattice to words in two variables

Abstract: The Chermak-Delgado measure of a subgroup H of a finite group G is defined as the product of the order of H with the order of the centralizer of H in G, i.e. $m_G(H) = |H|\,|C_G(H)|$, and the set of all subgroups with maximal Chermak-Delgado measure forms a lattice in G. Let $f(x,y)$ be a word in the alphabet $\{x,y,x^{-1},y^{-1}\}$ and H a subgroup of a group G. The following sets are subgroups of G: \begin{align*} F_1^{\ell}(G,H) &= \{a\in G\mid f(ag,h)=f(g,h)\ \forall g\in G,\forall h\in H\}\\ F_1^r(G,H) &= \{a\in G\mid f(ga,h) = f(g,h)\ \forall g\in G,\forall h\in H\}\\ F_2^{\ell}(G,H) &= \{a\in G\mid f(h,ag) = f(h,g)\ \forall g\in G,\forall h\in H\}\\ F_2^r(G,H) &= \{a\in G\mid f(h,ga) = f(h,g)\ \forall g\in G, \forall h\in H\} \end{align*} For the commutator word $f(x,y) = [x,y]$, we have \[F_1^{\ell}(H) = F_2^{\ell}(H) = C_G(H)\text{ and }

F_1^r(H)=F_2^r(H)= C_G(H^G).\] The Chermak-Delgado measure associated with the subgroup $F_i^t(G,H)$, with $i=1,2$ and $t=\ell,r$, is $m_G(H) = |H|\,|F_i^t(G,H)|$. The question arises for which words $f(x,y)$ the subgroups having maximal Chermak-Delgado measure form a lattice. We discuss the obstacles that the generalization encounters, and some situations in which they can be surmounted.

- **November 15, Combinatorics Seminar, 1:15-2:15**
 Christian Gaetz (Cornell University)
 1-skeleton posets of Bruhat interval polytopes

 Abstract: Bruhat interval polytopes are combinatorially interesting polytopes arising from total positivity and from certain toric varieties. I study the 1-skeleta of these polytopes, viewed as posets interpolating between weak order and Bruhat order. Interestingly, these posets turn out to be lattices and the polytopes, despite not necessarily being simple, have interesting h-vectors. I will give a criterion for determining when these polytopes are simple, or equivalently when generic torus orbit closures in Schubert varieties are smooth, solving a conjecture of Lee–Masuda.

- **November 15**
 Christian Gaetz (Cornell University)
 Stable characters from permutation patterns

 Abstract: We study the expected value (and higher moments) of the number of occurrences of a fixed permutation pattern on conjugacy classes of the symmetric group S_n. We prove that this virtual character stabilizes as n grows, so that there is a single polynomial computing these moments on any conjugacy class of any symmetric group. Our proof appears to be the first application of partition algebras to the study of permutation patterns. I'll also discuss partial progress towards a conjecture on when these virtual characters are genuine characters. This is joint work with Christopher Ryba and Laura Pierson.

- **November 22**
 Ryan McCulloch (Elmira College)
 On groups with few subgroups not in the Chermak-Delgado lattice

 Abstract: The Chermak-Delgado lattice of a finite group G, denoted $CD(G)$, is a modular sublattice of the lattice of subgroups of G. $CD(G)$ has nice properties, for example it is self-dual (and so if you turn the lattice upside down, you arrive at the same lattice). $CD(G)$ has been studied extensively in recent years. A recent question of Fasolă & Tărnăuceanu asks about subgroups not in $CD(G)$. Let $\delta(G) = |L(G)| - |CD(G)|$ where $L(G)$ denotes the lattice of all of the subgroups of a finite group G. Fasolă & Tărnăuceanu classify all finite groups G with $\delta(G) < 3$. We extend the classification for all finite groups G with $\delta(G) < 5$. We also obtain a classification for the non-nilpotent case when $\delta(G) = 5$. The non-nilpotent cases make extensive use of Sylow theory, and the p-group cases blend the computational with the theoretical. This is joint work with David Burrell and William Cocke.

- **November 29**
 Andrew Velasquez-Berroteran (Binghamton University)
 Equal Coverability of Finite Groups

 Abstract: A finite group G has an equal covering if it is a union of a collection of proper subgroups of

G that are all of the same order. This talk has two parts. The first part will present the work done in my undergraduate thesis at Adelphi University aimed at finding which finite groups under order 61, and which simple groups under order 100,000, have an equal covering. The second part will discuss the work done by my undergraduate advisor, Tuval Foguel, and his colleagues Alireza Moghaddamfar and Jack Smith. Their work this past summer expanded on my thesis, including a proof of the following claim I made in my thesis: No simple finite group has an equal covering.

- **December 6**
  Eilidh McKemmie (Rutgers University)
  Galois groups of random additive polynomials

  Abstract: The Galois group of an additive polynomial over a finite field is contained in a finite general linear group. We will discuss three different probability distributions on these polynomials, and estimate the probability that a random additive polynomial has a "large" Galois group. Our computations use a trick that gives us characteristic polynomials of elements of the Galois group, so we may use our knowledge of the maximal subgroups of GL(n,q). This is joint work with Lior Bary-Soroker and Alexei Entin.
