# The Algebra Seminar

Unless stated otherwise, the seminar meets Tuesdays in room WH-100E at 2:50 p.m. There will be refreshments served at 4:00 in room WH-102.

Organizers: Alex Feingold and Hung Tong-Viet

To receive announcements of seminar talks by email, please join the seminar's mailing list.

---

## Fall 2019

- **August 27**
  Organizational meeting

- **September 3**
  Casey Donoven (Binghamton University)
  ***Automata acting on Fractal Spaces***

  ***Abstract***: A self-similar set is a set that is a union of scaled copies of itself. Through iterated labeling of the $n$ copies, $n^2$ subcopies, and so on, we create a correspondence between infinite sequences over an n letter alphabet and points in the self-similar set. Automata act naturally on infinite sequence, and I will explore groups of homeomorphisms of semi-similar sets induced by automata. I will focus on two examples, the unit interval and Julia set associated to the map $z^2+i$. An important tool in the construction of the automata is the approximation of these self-similar sets as finite graphs.

- **September 10**
  Matt Evans (Binghamton University)
  ***BCK-algebras and generalized spectral spaces***

  ***Abstract***: Commutative BCK-algebras are the algebraic semantics of a non-classical logic. Mimicing the construction of the spectrum of a commutative ring (or Boolean algebra or distributive lattice), we can construct the spectrum of a commutative BCK-algebra. A topological space is called *spectral* if it is homeomorphic to the spectrum of some commutative ring, and *generalized spectral* if it is homeomorphic to the spectrum of a distributive lattice with 0. In this talk I will briefly discuss Hochster's characterization of spectral spaces, and then show that the spectrum of a commutative BCK-algebra is generalized spectral.

- **September 17**

Jonathan Doane (Binghamton University)
***Dualizing Kleene Algebras***

***Abstract***: It is well-known that the class of Boolean algebras is "generated" by the two element chain $F<T$ equipped with negation $\neg F:= T$, $\neg T:=F$. When we include an uncertainty element $F<U<T$, along with negation $\neg U: =U$, we generate the class of Kleene algebras. Of course, there is a famous correspondence between Boolean algebras and Boolean topological spaces, named Stone duality; this leads us to wonder if we can somehow represent Kleene algebras by topological spaces as well. In fact, Stone duality is but an application of a more general theory of dual equivalences between categories. In this talk, we will utilize this theory to construct a dual equivalence between the categories of Kleene algebras and certain topological spaces.

- **September 24**
  Cancelled

- **October 1**
  No Classes (University)
  ***Title***

  ***Abstract***: Abstract

- **October 8**
  Ben Brewster (Binghamton University)
  ***The collection of intersections of Sylow p-subgroups of G***

  ***Abstract***: Suppose $G$ is a finite group, $p$ is a prime integer which divides $|G|$. Brodkey's Theorem appeared in 1963. It says that if a Sylow $p$-subgroup of $G$ is abelian, then the intersection of all Sylow $p$-subgroups is the intersection of a pair of them. I began to wonder what the nature of the collections of intersections of all subsets of Sylow $p$-subgroups looked like. Clearly this is a meet semilattice under set inclusion. There are examples by Ito to show the minimal elements need not be intersections of two Sylow $p$-subgroups. During a sabbatical in 1989, I went to Tübingen, Germany where Peter Hauck and I collaborated to uncover some things about this collection of all intersections of subsets of Sylow $p$-subgroups. I will describe a few of the steps we used and hope to show some of main points on how and why some extra hypothesis is needed for some primes to obtain a resemblance of Brodkey's Theorem.

- **October 15**
  Fikreab Admasu (Binghamton University)
  ***Generating series for counting finite p-groups of class 2***

  ***Abstract***: In 2009, C. Voll computed the numbers $g(n, 2, 2)$ of nilpotent groups of order $n$, of class at most $2$ generated by at most $2$ generators, by giving an explicit formula for the Dirichlet generating function $\sum_{n=1}^{\infty} g(n, 2, 2)n^{-s}.$ Later in $2012$, Ahmad, Magidin and Morse gave a direct enumeration of such groups building on works of M. Bacon, L. Kappe, et al. We use their enumeration to provide a natural multivariable extension of the generating function counting such groups and as a result rederive Voll's explicit formula. Similar formulas or enumerations for finite groups of nilpotency class $2$ on more than $2$ generators or of at least class $3$ on $2$ or more generators is currently unknown.

- **October 22**
  Eran Crockett (Binghamton University)
  ***Finitely related clones***

  ***Abstract***: What is a clone? What does it mean for a clone to be finitely related? What are some examples of finitely related clones? What are some examples of non-finitely related clones? We answer these questions and more.

- **October 29**
  David Biddle (Binghamton University)
  ***Generating tuples of direct products of finite simple groups***

  ***Abstract***: For any group $G$ we can define the $n^{th}$ Eulerian function $\phi_n(G)$, to be the number of tuples in $G^n$ that generate $G$ and the rank of $G$ to be the smallest integer $d=d(G)$ so that $G$ has a generating set of size $d$. We will show that if we define the reduced Eulerian function to be $r_n(G):=\phi_n(G)/|\text{Aut}(G)|$, that for $S$ finite simple and $n \geq 2$, $\text{rank}(S^{r_n(S)})=n$ precisely. This has been famously used to show for instance that $\text{rank}((A_5)^{20})=3$ while $\text{rank}((A_5)^{19})=2$.

- **November 5**
  Luise Kappe (Binghamton University)
  ***A GAP-conjecture and its solution: isomorphism classes of capable special $p$-groups of rank 2***

  ***Abstract***: A group is said to be capable if it is a central quotient group and a $p$-group is special of rank 2 if its center is elementary abelian of rank 2 and equal to its commutator subgroup. In 1990, Heineken showed that if $G$ is a capable special $p$-group of rank 2, then $p^5 \leq |G| \leq p^7$. Over a decade ago we asked GAP to determine the number of isomorphism classes of capable special $p$-groups of rank 2 for small primes $p$. GAP told us that in these cases, the number of isomorphism classes of special $p$-groups of rank 2 grows with $p$. However, for the capable among them the number of isomorphism classes is independent of the prime $p$. Finally, we were able to show that what GAP conjectured is true for all primes $p$.

- **November 12**
  Dikran Karagueuzian (Binghamton University)
  ***Coalescence of Polynomials over Finite Fields***

  ***Abstract***: A polynomial over a finite field can be regarded as a map of the finite field to itself. The coalescence, or variance of the inverse image sizes, has been studied in connection with whether such maps are a good substitute for random maps. We are able to show that polynomial maps are not random in the sense that the coalescence must be an integer, in an asymptotic sense as the size of the finite field becomes large. This is based on joint work with Per Kurlberg.

- **November 19**
  Cancelled

- **November 26**
  Alex Feingold (Binghamton University)
  ***Monstrous Moonshine***

  ***Abstract***: This will be an introduction to the main ideas involved in the Conway-Norton monstrous moonshine, connections between the largest sporadic simple group, modular functions and vertex operator algebras.

- **December 3**
  No Meeting
  ***Title***

  ***Abstract***: Abstract

---

---