The Algebra Seminar

Unless stated otherwise, the seminar meets Tuesdays in room WH-100E at 2:50 p.m. There will be refreshments served at 4:00 in room WH-102.

Organizers: Alex Feingold and Hung Tong-Viet

To receive announcements of seminar talks by email, please join the seminar's mailing list.

Fall 2018

- **August 28**
  Organizational Meeting
  Title of Talk

  Abstract: Please come or contact the organizers if you are interested in giving a talk this semester or want to invite someone.

- **September 4**
  Casey Donoven (Binghamton University)
  Covering Number of Semigroups

  Abstract: A semigroup is a set S equipped with an associative operation. The covering number of a semigroup S is the minimum number of proper subsemigroups whose union is S. In this talk, I will introduce basic semigroup theory and some fundamental examples while proving the following theorem: If S is a finite semigroup that is not a group nor generated by a single element, then the covering number of S is 2. Similar questions have been studied for groups and loops. (Joint work with Luise-Charlotte Kappe and Marcin Mazur.)

- **September 11**
  No Classes (University)
  Title

  Abstract: Abstract text

- **September 18**

Joe Cyr (Binghamton University)
Semilattice Modes

Abstract: A mode A is an algebra which is idempotent and in which every operation is a homomorphism from the appropriate power of A to A. We will explore some results on a particular class of modes which are constructed from semilattices. In particular, we will look at the question of when is a semilattice mode subdirectly irreducible, both in general and in the particular case of when the mode has a single binary operation.

- **September 25**
Matt Evans (Binghamton University)
Spectral properties of involutory BCK-algebras

Abstract: BCK-algebras are algebraic structures arising from non-classical logic. This talk will focus primarily on the classes of commutative BCK-algebras and commutative involutory BCK-algebras. Particularly, I will discuss some basic ideal theory and spectral properties of such algebras, looking at differences between the bounded and unbounded cases.

- **October 2**
Mark Lewis (Kent State University)
Centers of centralizers and maximal abelian subgroups

Abstract: In this talk we consider the centers of the centralizers of elements in finite groups. We will then obtain a lower bound on the order of a maximal abelian subgroup in terms of the indices of the centralizers of elements and the orders of the centers of the centralizers of elements. We will use this to obtain a lower bound for maximal abelian subgroups of semi-extraspecial groups.

- **October 9**
Fernando Guzman (Binghamton University)
Polynomials Automorphisms of the Regular d-ary Tree

Abstract: We explore the question of when does a polynomial with integer coefficients induce an automorphism of the infinite regular d-ary tree. This is well-known for d=2, and there are some partial results for d prime. We extend the results to the case when d is square-free.

- **October 16**
Luise-Charlotte Kappe (Binghamton University)
A generalization of the Chermak-Delgado lattice to words in two variables

Abstract: The Chermak-Delgado measure of a subgroup H of a finite group G is defined as the product of the order of H with the order of the centralizer of H in G, $|H||C_G(H)|$, and the set of all subgroups with maximal Chermak-Delgado measure forms a dual sublattice of the subgroup lattice of G. In this talk we step back from centralizers and consider four types of centralizer-like subgroups, defined using general words in the alphabet $\{x, y, x^{-1}, y^{-1} \}$ instead of the specific commutator word. We show that this generalization results in four sublattices of the subgroup lattice of a finite group, some of which may be equal to one another depending on the word. We consider which properties of the Chermak-Delgado lattice generalize to the new lattices, and which properties are specialized in the Chermak-Delgado lattice. (This work is joint with Elizabeth Wilcox.)

- **October 23**
  Eran Crockett (Ripon College)
  The variety generated by the triangle

  Abstract: This talk will consist of a (quick) introduction to universal algebra where we focus on three topics: the definability of principal congruences, classifying subdirectly irreducibles, and determining the clone of term operations. We will attempt to understand these topics by focusing on two examples: the two-element semilattice and the three-element non-transitive tournament (a.k.a. the triangle).

- **October 30**
  Dan Rossi (Binghamton University)
  Brauer characters and fields of values

  Abstract: The complex irreducible characters $\text{Irr}(G)$ of a finite group G contain a lot of information about G itself. Recently, it has been realized that some of this information can still be captured if, instead of considering the entire set $\text{Irr}(G)$, one only considers those irreducible characters taking values in some suitable subfield of $\mathbb{C}$. This motivates the following definitions: $\text{Irr}_\mathbb{F}(G)$ is the subset of irreducible characters taking values in the subfield $\mathbb{F}\subseteq \mathbb{C}$; and $\text{Cl}_\mathbb{F}(G)$ is the set of conjugacy classes of G whose elements, when evaluated at every character of G, take values in $\mathbb{F}$. A result of Isaacs $\&$ Navarro says that $|\text{Irr}_\mathbb{F}(G)| = |\text{Irr}_\mathbb{F}(G/N)|$ and $|\text{Cl}_\mathbb{F}(G)| = |\text{Cl}_\mathbb{F}(G/N)|$ whenever $N\unlhd G$ contains no non-trivial $\mathbb{F}$-elements. For a prime p, the p-Brauer characters of G arise from representations of G over $\overline{\mathbb{F}}_p$. They provide a link between the representation theory of G in characteristic 0 and in characteristic p. Whenever one has a relationship involving characters and conjugacy classes G, it is natural to wonder if there is an analogous relationship between the p-Brauer characters and p-regular conjugacy classes. I will give some examples of the sorts of $\mathbb{F}$-generalizations alluded to in the first paragraph; introduce the basic notions of Brauer characters; and discuss a Brauer analogue of the Isaacs-Navarro result mentioned above.

- **November 6**
  Casey Donoven (Binghamton University)
  Covering Number of Semigroups (cont.)

  Abstract: I will continue my exploration of covering numbers of semigroups by considering specific classes of semigroups. A monoid is a semigroup with an identity. An inverse semigroup I is a semigroup such that for each element $a\in I$ there exists a unique element $a^{-1}\in I$ such that $aa^{-1}a=a$ and $a^{-1}aa^{-1}=a$. I will give a complete description of the covering number of monoids and inverse semigroups with respect to submonoids and inverse subsemigroups respectively (modulo the covering numbers of groups and semigroups). I will use Green's relations and other results to describe the structure of such semigroups.

- **November 13**
  Casey Donoven (Binghamton University)
  Fractal Subgroups of Profinite Groups

  Abstract: Often, we think of fractals as subsets of $\mathbb{R}^n$. Many definitions in fractal geometry can be generalized to any metric space, including groups equipped with a metric. In particular, the Hausdorff dimension and Box counting dimension can be defined on any metric space. Profinite groups can be equipped with a natural metric, under which we can discuss fractal properties. This will be an expository talk in which I define fractal dimensions and profinite groups. My goal is to set up the following question: Given two fractal subgroups of the automorphism group of the rooted infinite n-ary tree, what is the dimension of their intersection?

- **November 20**
  Hung Tong-Viet (Binghamton University)
  Conjugacy classes of p-elements and normal p-complements

  Abstract: The commuting probability $d(G)$ of a finite G (introduced by Erdős and Turán in 1968), is defined to be the probability that two randomly chosen elements of G commute. The commuting probability $d(G)$ is also called the commutativity degree of G. Erdős and Turán showed that $d(G)=k(G)/|G|$, where $k(G)$ is number of conjugacy classes of G. In 1973, W. H. Gustafson proved that $d(G)\leq 5/8$ for any non-abelian group G. Since then, there are numerous results concerning the structure of finite groups using various bounds on the commuting probability. In this talk, I will consider a p-local version of the commuting probability. Specifically, for a prime p, we define $d_p(G)$ to be the ratio $k_p(G)/|P|$, where $k_p(G)$ is the number of conjugacy classes of p-elements of G and P is a Sylow p-subgroup of G. Using the invariant $d_p(G)$, we obtain some new criteria for the existence of normal p-complements in finite groups.

- **November 27**
  Nicholas Gardner (Binghamton University)
  An Introduction to the Chermak-Delgado Lattice

  Abstract: For a subgroup H of a finite group $G,$ the Chermak-Delgado measure of H is defined as $m_{G}(H) := |H||C_{G}(H)|$. The subgroups of G with maximum Chermak-Delgado measure form a dual sublattice of the subgroup lattice of G. In this talk I will discuss some properties of such maximum-measure subgroups and calculate the Chermak-Delgado lattice for some classes of finite groups.

- **December 4**
  Speaker (University)
  Title

  Abstract: Abstract text

- Pre-2014 semesters
- Fall 2014
- Spring 2015
- Fall 2015
- Spring 2016
- Fall 2016
- Spring 2017
- Fall 2017
- Spring 2018