Problem 4 (due Monday, April 12)

a) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function such that $f(\sin x) = \sin f(x)$ for every $x \in \mathbb{R}$. Prove that if $f$ is not identically zero then $\lim_{x \to 0} \frac{f(x)}{x}$ exists and is equal to $1$ or $-1$.

b) Prove that there is a continuous function $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $f(\sin x) = \sin f(x)$ and $\lim_{x \to 0^+} \frac{f(x)}{x}$ does not exist.

Overview

Every other Monday (starting 02/15/21), we will post a problem to engage our mathematical community in the problem solving activity and to enjoy mathematics outside of the classroom. Students (both undergraduate and graduate) are particularly encouraged to participate as there is no better way to practice math than working on challenging problems. If you have a solution and want to be a part of it, e-mail your solution to Marcin Mazur (mazur@math.binghamton.edu) by the due date. We will post our solutions as well as novel solutions from the participants and record the names of those who've got the most number of solutions throughout each semester.

When you submit your solutions, please provide a detailed reasoning rather than just an answer. Also, please include some short info about yourself for our records.

Previous Problems and Solutions

- **Problem 3** Solved by Paul Barber, Yuqiao Huang, Prof. Vladislav Kargin, and Ashton Keith.

- **Problem 2** Only one solution was submitted, by Ashton Keith.

- **Problem 1** Solved by Chris Eppolito, Yuqiao Huang, Ashton Keith, Maxwell T Meyers, and Wei Yang.

- **Fall 2020**

- **Summer Challenge**

- **Spring 2020**