Problem 7 (due Monday, May 8)

5050 bats live in groups occupying some of the 5050 caves in the Magic Mountains (where bats live forever). Every night one bat from each occupied cave flies out. The bats meet and play together all night. At dawn they all go to one (the same) of the unoccupied caves to rest. Show that from some point on there will be the same number of bats out every night.

This problem in a slightly different wording is known as the {\bf Bulgarian solitaire}. We received three solutions. Prof. Vladislav Kargin submitted a beautiful solution, much simpler than our original solution (which reduces the problem to our Problem 6 this semester). The solutions by Aleksandr Aksenchuk and Krishnaraj Sambath are incomplete and contain some interesting partial observations, like the realization that the equality $5050=1+2+\ldots+100$ plays important role here and that from some day on we should always have one cave with exactly $i$ bats for $i=1,2,\ldots, 100$. For detailed solutions see the following link Solution. See also the following link for a nice discussion of this problem https://www.researchgate.net/publication/273158618_The_Bulgarian_solitaire_and_the_mathematics_around_it