Problem 5 (due on Monday, November 7)

Let $\cal F$ be the set of all functions $f:\mathbb R\longrightarrow \mathbb N$ from the real numbers to natural numbers. Prove that there exists a sequence of functions $f_1,f_2, f_3,\ldots$ in $\cal F$ such that for every finite set $A\subseteq \mathbb R$ and every $g\in \cal F$ there is i such that $g(a)=f_i(a)$ for every $a\in A$.

The problem was solved by Levi Axelrod and Ashton Keith. The solutions differ in details but follow similar idea. For a detailed solution and some applications to topology see the following link Solution.