

Problem 4 (due Monday, April 12)

- a) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function such that $f(\sin x) = \sin f(x)$ for every $x \in \mathbb{R}$. Prove that if f is not identically zero then $\lim_{x \rightarrow 0} \frac{f(x)}{x}$ exists and is equal to 1 or -1 .
- b) Prove that there is a continuous function $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $f(\sin x) = \sin f(x)$ and $\lim_{x \rightarrow 0^+} \frac{f(x)}{x}$ does not exist.

Two solutions were submitted: by Paul Barber and Ashton Keith. Neither one is complete. Ashton attempts to solve part a) under additional assumption that f is continuous at 0. While his solution has some gaps, the ideas are very nice indeed and they can be improved to a complete solution (under the additional assumption). For more details and to see complete solutions see the following link [Solution](#).

From:

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