

Problem 4 (due Monday, April 12)

- a) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function such that  $f'(x) = \sin f(x)$  for every  $x \in \mathbb{R}$ . Prove that if  $f$  is not identically zero then  $\lim_{x \rightarrow 0} \frac{f(x)}{x}$  exists and is equal to  $1$  or  $-1$ .
- b) Prove that there is a continuous function  $f: \mathbb{R} \rightarrow \mathbb{R}$  such that  $f'(x) = \sin f(x)$  and  $\lim_{x \rightarrow 0^+} \frac{f(x)}{x}$  does not exist.

Two solutions were submitted: by Paul Barber and Ashton Keith. Neither one is complete. Ashton attempts to solve part a) under additional assumption that  $f$  is continuous at 0. While his solution has some gaps, the ideas are very nice indeed and they can be improved to a complete solution (under the additional assumption). For more details and to see complete solutions see the following link [Solution](#).

From:

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