Problem 4 (due Monday, October 25)

a) Let $x_1,\ldots, x_n$ be real numbers. Prove that \[\sum_{i=1}^n\sum_{j=1}^n \frac{\sin(x_i-x_j)}{x_i-x_j}\geq \sum_{i=1}^n\sum_{j=1}^n \frac{\sin(x_i+x_j)}{x_i+x_j} \] with the convention that $\displaystyle \frac{\sin x}{x}=1$ when $x=0$.

b) Compute $\displaystyle \int_0^1 \sin 2x\sin 5x \ \text{d}x$.

The problem was solved by Ashton Keith. Ashton's solution is similar to our solution. For details see the following link Solution.