Problem 3 (due Monday, October 9)

Given a sequence $a_1, a_2, dots, a_n \ of \ n \ real numbers we construct a new sequence of$ $numbers as follows: first we set <math>b_i=\max(a_i,a_{i+1})\$ for i=1, dots, n-1. Then we choose randomly one index $ii\$ and add $11 \$ to b_i . This is our new sequence. After repeating this operation n-1 times we arrive at a single number A. Prove that if $a_1+\ldots$ $+a_n=0$, then $A \ge 1$.

Here \$\max(a,b)\$ denotes the larger of the numbers \$a,b\$.

We did not receive any solutions. The key idea for our solution is to observe that the quantity $2^{a_1}+1$ dots + 2^{a_n} is a monovariant for our operation on sequences, i.e. that this quantity computed for the new sequence is larger or equal than the quantity for the original sequence. It follows that $2^{A} = 2^{a_1} + 1$ dots + 2^{a_n} . By the AMGM inequality (the arithmetic mean is always greater or equal than the geometric mean), we have $2^{a_1} + 1$ dots + 2^{a_n} , hence 2^{a_n} . For a detailed solution and some additional discussion see the following link Solution.

From: https://www2.math.binghamton.edu/ - Department of Mathematics and Statistics, Binghamton University

×

Permanent link: https://www2.math.binghamton.edu/p/pow/problem3f23

Last update: 2023/10/10 21:23