

## Problem 3 (due Monday, October 9)

Given a sequence  $a_1, a_2, \dots, a_n$  of  $n$  real numbers we construct a new sequence of  $n-1$  numbers as follows: first we set  $b_i = \max(a_i, a_{i+1})$  for  $i=1, \dots, n-1$ . Then we choose randomly one index  $i$  and add  $1$  to  $b_i$ . This is our new sequence. After repeating this operation  $n-1$  times we arrive at a single number  $A$ . Prove that if  $a_1 + \dots + a_n = 0$ , then  $A \geq \log_2 n$ .

Here  $\max(a, b)$  denotes the larger of the numbers  $a, b$ .

We did not receive any solutions. The key idea for our solution is to observe that the quantity  $2^{a_1} + \dots + 2^{a_n}$  is a monovariant for our operation on sequences, i.e. that this quantity computed for the new sequence is larger or equal than the quantity for the original sequence. It follows that  $2^A \geq 2^{a_1} + \dots + 2^{a_n}$ . By the AMGM inequality (the arithmetic mean is always greater or equal than the geometric mean), we have  $2^{a_1} + \dots + 2^{a_n} \geq n$ , hence  $A \geq \log_2 n$ . For a detailed solution and some additional discussion see the following link [Solution](#).

From:

<https://www2.math.binghamton.edu/> - **Department of Mathematics and Statistics, Binghamton University**

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