Problem 3 (due on Monday, October 10)

a) Is there a function $f:\mathbb R\longrightarrow \mathbb R$ such that \[\frac{f(x)+f(y)}{2}\geq f\left(\frac{x+y}{2}\right)+ \sin^2(x-y)\] for all $x,y\in \mathbb R$?

b) Is there a function $f:\mathbb R\longrightarrow \mathbb R$ such that \[\frac{f(x)+f(y)}{2}\geq f\left(\frac{x+y}{2}\right)+ \sin|x-y|\] for all $x,y\in \mathbb R$?

The answer to a) is positive, for example $f(x)=4x^2$ has the property. The answer to b) is negative. We received only one solution, from Prof. Vladislaw Kargin, who solved a) and b) under additional assumption about f (essentially that f is continuous). For a detailed solution and some related material see the following link Solution.