Problem 2 (due Monday, September 22)

Find all functions $F:\mathbb R^2\longrightarrow \mathbb R$ such that

(i) If $ABCD$ is any rectangle on the plane $\mathbb R^2$ then $F(A)+F(C)=F(B)+F(D)$;

(ii) The second order partial derivatives $\displaystyle \frac{\partial^2 F}{\partial x \partial x}$, $\displaystyle \frac{\partial^2 F}{\partial x \partial y}$, $\displaystyle \frac{\partial^2 F}{\partial y \partial x}$, $\displaystyle \frac{\partial^2 F}{\partial y \partial y}$ exist and are continuous on $\mathbb R^2$ (this means that F is of class C^2);

(iii) $F(0,0)=0$, $F(1,0)=1=F(0,1)$, $\displaystyle \frac{\partial F}{\partial x}(0,0)=0$.

The problem was solved by Gerald Marchesi, Josiah Moltz, and Mathew Wolak. The only function which satisfies the conditions of the problem is $F(x,y)=x^2+y^2$. For detailed solutions and additional discussion see the following link Solution.