

Problem 2 (due Monday, September 22)

Find all functions $F: \mathbb{R}^2 \rightarrow \mathbb{R}$ such that

- (i) If $ABCD$ is any rectangle on the plane $\mathbb{R}^2$ then $F(A)+F(C)=F(B)+F(D)$;
- (ii) The second order partial derivatives $\frac{\partial^2 F}{\partial x \partial x}$, $\frac{\partial^2 F}{\partial x \partial y}$, $\frac{\partial^2 F}{\partial y \partial x}$, $\frac{\partial^2 F}{\partial y \partial y}$ exist and are continuous on $\mathbb{R}^2$ (this means that F is of class C^2);
- (iii) $F(0,0)=0$, $F(1,0)=1=F(0,1)$, $\frac{\partial F}{\partial x}(0,0)=0$.

The problem was solved by Gerald Marchesi, Josiah Moltz, and Mathew Wolak. The only function which satisfies the conditions of the problem is $F(x,y)=x^2+y^2$. For detailed solutions and additional discussion see the following link [Solution](#).

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