Problem 2 (due Monday, March 2)

Recall that the smallest integer greater or equal than a given real number x is denoted by $\left \lceil x \right\rceil $ and called the $ceiling$ of x. Let p be a prime number and $1\leq a<p$ an integer. Prove that the number

$$ \left \lceil \left(a^{p-1}-1 \right)^{\frac{p}{p-1}} \right\rceil $$ is divisible by p. What can you say when $a=p$?

This problem was solved by only one participant: Yuqiao Huang. The submitted solution does not discuss the case when $a=p$. For $1\leq a\leq p-1$, the solution claims correctly that $\left \lceil \left(a^{p-1}-1 \right)^{\frac{p}{p-1}} \right\rceil= a^p-a$, hence the result follows from Fermat's Little Theorem. In order to justify the claim, the solver proves that $$ a^p-a-1< \left(a^{p-1}-1 \right)^{\frac{p}{p-1}}\leq a^p-a.$$ The proof of the right hand side inequality is fairly simple; the submitted proof of the left hand side inequality is rather long and complicated, and we will not reproduce it here. To see a detailed solution click the following link Solution

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http://www2.math.binghamton.edu/p/pow/problem2

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