

Problem 2 (due Monday, March 2)

Recall that the smallest integer greater or equal than a given real number x is denoted by $\lceil x \rceil$ and called the ceiling of x . Let p be a prime number and $1 \leq a < p$ an integer. Prove that the number

$\lceil (a^{p-1} - 1)^{\frac{p}{p-1}} \rceil$ is divisible by p . What can you say when $a = p$?

This problem was solved by only one participant: Yuqiao Huang. The submitted solution does not discuss the case when $a = p$. For $1 \leq a \leq p-1$, the solution claims correctly that $\lceil (a^{p-1} - 1)^{\frac{p}{p-1}} \rceil = a^p - a$, hence the result follows from Fermat's Little Theorem. In order to justify the claim, the solver proves that $a^p - a - 1 < (a^{p-1} - 1)^{\frac{p}{p-1}} \leq a^p - a$. The proof of the right hand side inequality is fairly simple; the submitted proof of the left hand side inequality is rather long and complicated, and we will not reproduce it here. To see a detailed solution click the following link [Solution](#)

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Last update: **2020/03/02 22:55**

