Problem 1 (due Monday, February 10)

For a positive integer n, let $s(n)$ be the square of the sum of the decimal digits of n. For example, $s(134)=(1+3+4)^2=64$. Find all n such that $n=s^k(n)$ for some k, where $s^k=s\circ s\circ \ldots \circ s$ is the composition of s with itself k-times.

We received a solution from David Biddle, ChatGPT via Prof. Vladislav Kargin, Colin McCann jointly with Michael Mancuso, Maxwell Meyers, Ethan Tong. One of the solutions was only partial, the others all followed essentially the same approach as our in-house solution. The answer is that there are only 4 numbers n satisfying the conditions: $1,81$ when $k=1$ and $169,256$ when $k=2$. The idea behind the solution is to note that there is m such that $s(n)<n$ for all $n> m$. Then it suffices to check for solutions among squares in the set $\{1,2,\ldots, m\}$, which can be done by hand (if m is small enough) or with a help of a computer (this is what ChatGPT produced without specifying m; actually $m=400$ works). For details see the following link Solution.