Problem 1 (due Monday, March 1)

We say that a vector in $\mathbb R^3$ is ${\rm positive}$ (${\rm negative}$) if all its coordinates are positive (resp. negative). Let v_1,v_2,v_3,v_4 be vectors in $\mathbb R^3$ such that the sum of any two of these vectors is either positive or negative. Prove that at least one of the vectors $v_1,v_2,v_3,v_4, v_1+v_2+v_3+v_4$ is either positive or negative.

The problem was solved by Chris Eppolito, Yuqiao Huang, Ashton Keith, Maxwell T Meyers, and Wei Yang. One solver provided an (almost complete) solution in which it was observed that for vectors in $\mathbb R^3$ actually one of the vectors v_1,v_2,v_3,v_4 is either positive or negative. The other solvers provided complete solutions which work for vectors in $\mathbb R^n$ for any n. Detailed solutions are discussed in the following link Solution.