Problem 1 (due Monday, September 11)

A continuous function $f:\mathbb R\longrightarrow \mathbb R$ has the following property: \[f(x)\cdot f(f(x))=1\ \text{for every}\ x\in\mathbb R.\] Knowing that the largest value of f is e, prove that \[3+e^{-2}<\int_0^{2e}f(t)dt<3+e^2.\] Show that these bounds are best possible. Here $e=2.7128\ldots$ is the base of natural logarithms.

We received solutions from Sasha Aksenchuk, Prof. Vladislav Kargin, Mithun Padinhare Veettil, and Daniel J. Riley (Tufts U.). All solvers provided a correct argument for the inequalities \[3+e^{-2}\leq\int_0^{2e}f(t)dt\leq 3+e^2.\] The justification that the above inequalities are strict was usually not provided in sufficient detail and some solutions did not provide sufficiently detailed justification that the bounds are best possible. For a detailed solution see the following link Solution.

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