Discrete Maths Section 1 (Spring 2020)

This is the official website of math314-01-s20.[1] Please read our syllabus...

Due to the COVID-19 pandemic, our course has gone entirely online. Please carefully read the updated syllabus and the site below.

THIS PAGE NO LONGER RECEIVES UPDATES

General Information

Meetings: Meetings take place during our old class time (08:00 - 09:30 on MWF).

Office Hours: By appointment on Zoom. Appointments should be made for reasonable times of the day ("reasonable" as defined by me).

Textbook: *Discrete Mathematics and its Applications (8e)* by Kenneth Rosen.

Grading: See the course syllabus for a grade distribution.

Content: Propositional logic, methods of proof, naive set theory, functions and relations, induction and recursion, counting, and basic graph theory.

Content Delivery

Videos: I will upload videos to replace some lectures. You are to watch these videos, take notes, and answer any questions posed in them.[2]

Notes: I will sometimes post PDFs for you to read. These are assigned reading; take notes on these PDFs, as they are fair game for homework and the final exam.

Readings: I will still assign reading from the textbook.

Problem Sessions: Roughly twice a week, I will host a problem session on Zoom during our old class meeting time. You are expected to attend these sessions.

Homework

Here is a quick look at what you will need to submit homework for the remainder of the course.

Submission

Homework is to be submitted as a PDF through Gradescope; the company has made their service free-to-use for this semester. See the Gradescope guide for students and watch the Gradescope video on submitting homework.

I will not accept homework in any form other than PDF.

I will not accept homework via email. Use Gradescope.

Miscellany

I offer you 1 homework bonus point per homework turned in using the LaTeX typesetting language; you must typeset exercises legibly. I wrote a short PDF on how to use LaTeX; you can use its source code as a startup template. The easiest way to get started is to make an account on Overleaf so you can view and compile .tex files; if you end up using LaTeX more frequently, you will probably want a different setup.

If you choose not to learn LaTeX, you can use a PDF scanning application to submit your homework. I think "Tiny Scanner" is nice; you can download it on your phone with either Android or iOS.

Here are documents on basic proof techniques and proof-writing style for your reference.

Participation

As a portion of your grade, you must contribute to our class notes. This can be done in one of the following two ways.

- **Notes**: Typing a portion of our notes from lectures (either one or two lectures).
- **Project**: Typing a short report on some topic related to our course.

All contributions must be approved by me in advance. The typing can be done either as plain-text or in $\LaTeX{}$; in either case students must discuss formatting with me before beginning their work.

Schedule

22 January 2020 W

- Review Syllabus
- Basic Logic and Sets
  - Propositions
  - Logical Connectives
- **Homework**:

- Read textbook sections 1.1, 1.3.1 - 1.3.5

24 January 2020 F

- Basic Logic and Sets
  - Translation of Sentences
  - Logical Equivalence
- **Homework**:
  - Read textbook sections 1.4 - 1.5

27 January 2020 M

- Basic Logic and Sets
  - Predicates and Quantifiers
- **Homework**:
  - Read textbook sections 2.1 - 2.2 and 1.6

29 January 2020 W

- Basic Logic and Sets
  - Sets and Set Operations
  - Rules of Inference
- **Homework**:
  - Read textbook section 1.6
  - Think about the relationship between the logic we know and the basic operations of set theory.

31 January 2020 F

- Basic Logic and Sets
  - Natural Deduction
- **Homework**:
  - Complete these exercises (Due: 10 February 2020)
  - Enjoy the Superb Owl! (more information here if you're unfamiliar)

3 February 2020 M

- Elementary Properties of Divisibility
  - Some careful mathematical proofs
- **Homework**:
  - Read textbook section 4.3; pay special attention to sections 4.3.2, 4.3.4, and 4.3.7-4.3.8.
  - Attempt problems from the textbook.
- *Add\Drop Deadline is TODAY!*

5 February 2020 W

- Modular Arithmetic
  - Constructed $\mathbb{Z}/n\mathbb{Z}$
  - Studied more basic properties of divisibility
- **Homework**:
  - Read textbook section 4.4; pay special attention to sections 4.4.2 and 4.4.5.
  - Attempt problems from the textbook.

7 February 2020 F

- sNOw Classes
  - Homework is still due on Monday.
  - Also still do the reading for Monday.

10 February 2020 M

- Greatest Common Divisor
  - Euclid's Algorithm
- Modular Arithmetic
  - Units in $\mathbb{Z}/n\mathbb{Z}$
  - Solving Equations in $\mathbb{Z}/n\mathbb{Z}$
- **Homework**:
  - Collected Homework 1
  - **Due Wednesday**: Prove the following proposition.
    - Let $a \in \mathbb{Z}$ and $m \in \mathbb{Z}^+$. Integer a is a unit modulo m if and only if $\gcd(a, m) = 1$.
  - Attempt problems from the textbook.

12 February 2020 W

- Primes and Factorization
  - Prime numbers and prime sieves
  - Fundamental Theorem of Arithmetic
- **Homework**:
  - Read textbook section 4.6
  - Watch this video from KhanAcademy.org.
  - Attempt problems from the textbook.

14 February 2020 F

- Application:
  - RSA Cryptography

    - RSA Problems
- **Homework**:
    - Read textbook section 5.1.
    - Attempt problems from the textbook.

17 February 2020 M

- Mathematical Induction
    - Introduction (with dominoes!)
    - Many simple examples
- **Homework**:
    - Attempt problems from the textbook.

19 February 2020 W

- Mathematical Induction
    - Fibonacci Numbers
    - More proofs by induction
- **Homework**:
    - Attempt problems from the textbook.

21 February 2020 F

- Review Session for Exam 1
    - Student questions only
- **Homework**: Study!!!

24 February 2020 M

- **Midterm Exam 1** (In Class)

26 February 2020 W

- Mathematical Induction
    - Review of Weak Induction
    - Strong Induction
- **Homework**:
    - Read textbook section 5.2
    - Watch this video on strong induction examples.
    - Attempt problems from the textbook.

28 February 2020 F

- Mathematical Induction
  - More Strong Induction
  - The Well Ordering Principle
- **Homework**:
  - **Due Monday**: Find (with proof!) all integers of the form $n = 5s + 9t$ for some $s, t \in \mathbb{N}$.
  - Read textbook section 5.3
  - Watch this video on comparing weak induction, strong induction, and well ordering (you can stop watching around the 5:45 marker).
  - Attempt problems from the textbook.

2 March 2020 M

- Recursion
  - Some interesting relationships among on Fibonacci numbers
  - *NB*: Recursion is an extremely important concept; much of what we do in the rest of the course requires understanding this fundamental topic…
- **Homework**:
  - **Due Wednesday**: Prove that every Fibonacci number with odd index (i.e. f_{2k+1} for $k \in \mathbb{N}$) can be expressed as a sum of squares of Fibonacci numbers.
  - Read textbook section 6.1
  - Attempt problems from the textbook.

4 March 2020 W

- Enumeration
  - Cardinality of Sets
  - Basic Counting Techniques
    - *Product Principle*: If A and B are finite sets, then $\#(A \times B) = \#A \cdot \#B$.
    - *Sum Principle*: If A and B are disjoint finite sets, then $\#(A \cup B) = \#A + \#B$.
      - *NB*: Take note that A and B must be disjoint!
    - *Correspondence Principle*: Let A and B be finite sets. If there is a rule of assignment $R \subseteq A \times B$ such that both $A = \{a : (a, b) \in R \text{ for some } b \in B\}$ and $\#\{b : (a, b) \in R\} = 1$ for all $a \in A$, and $B = \{b : (a, b) \in R \text{ for some } a \in A\}$ and $\#\{a : (a, b) \in R\} = 1$ for all $b \in B$, then $\#A = \#B$.
  - Many Examples
- **Homework**:
  - Read textbook sections 6.3 and 6.4
  - Attempt problems from the textbook.

6 March 2020 F

- No Class (Winter Break)

9 March 2020 M

- Enumeration
  - Binomial Coefficients
    - Proved several identities via counting
  - *Binomial Theorem*: Let $n \in \mathbb{N}$. For all $x, y \in \mathbb{R}$ we have $(x + y)^n = \sum_{k = 0}^n \binom{n}{k} x^k y^{n-k}$.
- **Homework**:
  - Read section 8.5 (*NB*: Yes, I mean "8.5"; it is not a typo)
  - **Due Wednesday** Give two proofs that for all $n \in \mathbb{Z}^+$ we have $\sum_{k = 0}^n (-1)^k \binom{n}{k} = 0$.
    - Use the Binomial Theorem.
    - Use a direct counting argument (*Hint*: move the negative terms to the other side before you start counting).
  - Attempt problems from the textbook.

11 March 2020 W

- Enumeration
  - *Inclusion-Exclusion Principle*: For all finite sets A and B we have $\#(A \cup B) = \#A + \#B - \#(A \cap B)$.
    - *NB*: Compare this with the Sum Principle…
- **Homework**:
  - Attempt problems from the textbook.

13 March 2020 F

- Enumeration
  - *NB*: This lecture taught over Zoom.
  - Problem session on enumeration.
- **Homework**:
  - Attempt problems from the textbook.

Note on Coronavirus Changes

In response to the COVID-19 pandemic, the remainder of the course went online. I have grouped the remainder of the semester by week (because this is more natural for an online course).

Old Lecture Notes

Here are lecture notes taken and plain-text typed by students–with editing and further LaTeX formatting done by me–for lectures before the class went online.

- Lecture Notes: Divisibility.
- Lecture Notes: Modular Arithmetic.
- Lecture Notes: Weak Induction.
- Lecture Notes: Strong Induction.
- Lecture Notes: Primes and Factorization.
- Lecture Notes: Fibonacci Numbers.
- Lecture Notes: Counting Techniques.
- Lecture Notes: Binomial Coefficients.

Week 9 (15 March 2020 -- 22 March 2020)

- **Topics**: Enumeration
  - Pigeonhole Principle and Generalized Pigeonhole Principle
    - Textbook section 6.2.
    - Lecture notes on the Pigeonhole Principle.
    - Video on the Generalized Pigeonhole Principle.
    - Lecture notes on the Generalized Pigeonhole Principle.
  - Anagrams
    - Textbook section 6.5.
    - Lecture notes.
    - Video on counting anagrams.
  - Counting Poker Hands (for practice; this is a different kind of counting cards...).
- **Homework**:
  - Submit a PDF to Gradescope with the text "This is a test homework. I took this opportunity to familiarize myself with Gradescope and making PDF submissions." (Due 23 March 2020)
    - See the homework section for more information.
  - Complete these counting problems (Due 27 March 2020).

Week 10 (22 March 2020 -- 28 March 2020)

- **Topics**: Functions and Relations.
  - Introduction to Relations
    - Textbook sections 9.1 and 9.3.
    - Lecture notes.
    - Video on relations: reflexive, symmetric, and transitive (PS. this guy's videos are pretty great!).
  - Equivalence Relations and Closures
    - Textbook sections 9.5 and 9.4.
    - Lecture notes.
    - Video on equivalence relations (this YouTube channel is also pretty nice!).
  - Functions

        - Textbook section 2.3.
        - Notes on functions and relations.
        - Video on injective functions.
        - Video on surjective functions.
- **Homework**:
    - Complete these homework problems (Due 13 April 2020).

Week 11 (29 March 2020 -- 4 April 2020)

- **Topics**: Graph Theory.
    - Introduction to Graphs
        - Textbook sections 10.1 – 10.3 (just for reference, see the lecture first).
        - Lecture Notes: Introduction to Graphs.
        - Lecture Notes: Proof of the Handshake Lemma.
        - Lecture Notes: Subgraphs and Counting Subgraphs of K_n.
    - Connection in Graphs
        - Textbook section 10.4.
        - Lecture Notes: Graph Connection.

Spring Break (5 April 2020 -- 11 April 2020)

- **No Scheduled Meetings**.

Week 12 (12 April 2020 -- 18 April 2020)

- **Topics**: Graph Theory
    - Eulerian and Hamiltonian Graphs
        - Textbook section 10.5.
        - Lecture Notes on Eulerian and Hamiltonian Graphs.
    - Graph Coloring
        - Textbook section 10.8.
        - Lecture Notes on Graph Coloring.

Week 13 (19 April 2020 -- 25 April 2020)

- **Topics**: Trees
    - Characterizing Trees
        - Textbook section 11.1.
        - Lecture Notes: Characterizing Trees.
    - Spanning Trees
        - Textbook sections 11.4 and 11.5.
        - Lecture Notes: Spanning Trees.
    - Storing Labeled Trees
        - Lecture Notes: Pruefer Code.

- **Homework**:
  - Complete these graph theory problems (Due 8 May 2020).
  - *Optional*: Implement the Pruefer code algorithm and its inverse in your favorite programming language.

Week 14 (26 April 2020 -- 2 May 2020)

- **Topics**: Models of Computation
  - Finite State Machines
    - Textbook section 13.2.
    - Lecture Notes.
  - Deterministic Finite State Automata
    - Textbook section 13.3.
    - Lecture Notes.
  - Noneterministic Finite State Automata
    - Textbook section 13.4.
    - Lecture Notes.

Week 15 (3 May 2020 -- 9 May 2020)

- **Review for Final**.

FINAL EXAM

Due to the COVID-19 pandemic, final exams will be administered orally over Zoom.

Here a list of topics to guide your studying for your oral final.

[1] If you have an idea to improve this space, please email eppolito-at-math-dot-binghamton-dot-edu with your suggestion; I would like this space to be as useful to students as possible...
[2] If you have trouble viewing the videos through Safari, you can either use another browser or try this solution from reddit.