2025/09/14 08:55 1/2 Homework

Math 504 - Homework

- LaTeX-ed solutions are encouraged and appreciated.
- If you use LaTeX, hand-in a printed version of your homework.
- You are encouraged to discuss homework problems with classmates, but such discussions should NOT include the exchange or written material.
- Writing of homework problems should be done on an individual basis.
- Outside references for material used in the solution of homework problems should be fully disclosed.
- References to results from the textbook and/or class notes should also be included.
- The following lists should be considered partial and tentative lists until the word complete appears next to it.
- Use 8.5in x 11in paper with smooth borders. Write your name on top of each page. Staple all pages.

\$\newcommand{\aut}{\textrm{Aut}} \newcommand{\end}{\textrm{End}} \newcommand{\sub}{\textrm{Sub}} \newcommand{\join} {\vee} \newcommand{\join} {\bigyee} \newcommand{\meet} {\wedge} \newcommand{\bigmeet}{\bigwedge} \newcommand{\normaleq} {\unlhd} \newcommand{\normal}{\lhd} \newcommand{\union}{\cup} \newcommand{\intersection}{\cap} \newcommand{\bigunion}{\bigcup} $\label{linear_loss} $$\operatorname{\bigcap} \newcommand {\sq}[2][\]_{\sqrt[#1]_{#2}_} $$$ $\label{thm:linear_command_linear_c$ \newcommand{\imp}{\Rightarrow} \newcommand{\rimp}{\Leftarrow} \newcommand{\pinfty}{1/p^\infty} \newcommand{\calT} \mathcal{T} \newcommand{\qal} \textrm{Gal} \newcommand{\isom}{\approx} \renewcommand{\hom}{\textrm{Hom}} \$

Problem Set 9 Due 05/05/2020 (complete)

- 1. Prove that a finite group \$G\$ is solvable iff there is a finite sequence of subgroups \[1=H_0\leq H_1 \leq \cdots \leq H_{n-1} \leq H_n=G \] such that each \$H_i\normaleq H_{i+1}\$ and \$H_{i+1}/H_i\$ is cyclic. Show, with a counterexample, that this equivalence does not hold in general for arbitrary groups.
- 2. Show that the class of solvable groups is not closed under arbitrary products.
- 3. (Optional) Redo Exercise 4.6.1 in the class notes (page 102)
- 4. Let p be prime, and $G\leq S_p$. Show that if G contains a p-cycle and a transposition, the $G=S_p$.

Problem Set 8 Due 04/28/2020 (complete)

- 1. Prove Theorem 4.24.1,2 in the class notes (page 90).
- 2. Exercise 4.6.1 in the class notes (page 101)
- 3. Let \$K\$ and \$L\$ be fields. Show that the set \$\hom(K,L)\$ of all homomorphisms from \$K\$ to \$L\$, is linearly independent over \$L\$ as a subset of the vector space \$L^K\$ of all functions from \$K\$ to \$L\$. In particular \$\aut(K)\$ is linearly independent over \$K\$.

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4. Let \$F/K\$ be a finite extension, and \$L/K\$ its normal closure. Show that \$L/K\$ is also a finite extension. Hint: if you write \$E=K(\alpha_1,\dots,\alpha_n)\$, and let \$f_i(x)=\min_K(\alpha_i)\$, show that \$L\$ is the splitting field of the set \$A=\{f 1(x),\dots,f n(x)\}\$.

Problem Set 7 Due 04/16/2020 (complete)

- 1. Prove or disprove: all cyclotomic polynomials have all their coefficients in \$\{0,\pm 1\}\$.
- 3. Let \$P\$ be a locally finite poset. For \$y\neq x\in P\$, show that \[\sum_{y\leq \ul\{z}\leq x}\mu(z,x)=0 \] Hint: Fix \$y\in P\$, and then use induction on the Artinian poset \[\{u\in P\mid u > y\}.\]
- 4. Show that the sequence of coefficients of the cyclotomic polynomial $\pi(x)$, for $\pi(x)$, i.e. if $\pi(x)=\sum_{i=0}^{\pi(x)}$, if $\pi(x)=\sum_{i=0}^{\pi(x)}$.

Earlier Homework

From:

 $https://www2.math.binghamton.edu/-\textbf{Department of Mathematics and Statistics, Binghamton}\\ \textbf{University}$

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