Math 504 - Homework

- LaTeX-ed solutions are encouraged and appreciated.
- If you use LaTeX, hand-in a printed version of your homework.
- You are encouraged to discuss homework problems with classmates, but such discussions should NOT include the exchange or written material.
- Writing of homework problems should be done on an individual basis.
- Outside references for material used in the solution of homework problems should be fully disclosed.
- References to results from the textbook and/or class notes should also be included.
- The following lists should be considered partial and tentative lists until the word complete appears next to it.
- Use 8.5in x 11in paper with smooth borders. Write your name on top of each page. Staple all pages.

$\newcommand{\aut}{\textrm{Aut}} \newcommand{\end}{\textrm{End}} \newcommand{\sub}{\textrm{Sub}} \newcommand{\min}{\textrm{min}} \newcommand{\lub}{\textrm{l.u.b.}} \newcommand{\glb}{\textrm{g.l.b.}} \newcommand{\join}{\vee} \newcommand{\bigjoin}{\bigvee} \newcommand{\meet}{\wedge} \newcommand{\bigmeet}{\bigwedge} \newcommand{\normaleq}{\unlhd} \newcommand{\normal}{\lhd} \newcommand{\union}{\cup} \newcommand{\intersection}{\cap} \newcommand{\bigunion}{\bigcup} \newcommand{\bigintersection}{\bigcap} \newcommand{\sq}[2][\ ]{\sqrt[#1]{#2\,}} \newcommand{\pbr}[1]{\langle #1\rangle} \newcommand{\ds}{\displaystyle} \newcommand{\C}{\mathbb{C}} \newcommand{\R}{\mathbb{R}} \newcommand{\Q}{\mathbb{Q}} \newcommand{\Z}{\mathbb{Z}} \newcommand{\N}{\mathbb{N}} \newcommand{\A}{\mathbb{A}} \newcommand{\F}{\mathbb{F}} \newcommand{\T}{\mathbb{T}} \newcommand{\ol}[1]{\overline{#1}} \newcommand{\ul}[1]{\underline{#1}} \newcommand{\imp}{\Rightarrow} \newcommand{\rimp}{\Leftarrow} \newcommand{\pinfty}{1/p^\infty} \newcommand{\power}{\mathcal{P}} \newcommand{\calL}{\mathcal{L}} \newcommand{\calC}{\mathcal{C}} \newcommand{\calN}{\mathcal{N}} \newcommand{\calB}{\mathcal{B}} \newcommand{\calF}{\mathcal{F}} \newcommand{\calR}{\mathcal{R}} \newcommand{\calS}{\mathcal{S}} \newcommand{\calU}{\mathcal{U}} \newcommand{\calT}{\mathcal{T}} \newcommand{\gal}{\textrm{Gal}} \newcommand{\isom}{\approx} \renewcommand{\hom}{\textrm{Hom}} $

Problem Set 13 Due 05/07/2018 (complete)

1. Let K and L be fields. Show that the set $\hom(K,L)$ of all homomorphisms from K to L, is linearly independent over L. In particular $\aut(K)$ is linearly independent over K.
2. Prove that a finite group G is solvable iff there is a finite sequence of subgroups \[1=H_0\leq H_1 \leq \cdots \leq H_{n-1} \leq H_n=G \] such that each $H_i\normaleq H_{i+1}$ and H_{i+1}/H_i is cyclic. Show, with a counterexample, that this equivalence does not hold in general for arbitrary groups.
3. Define: an angle θ is constructible if there are two constructible straight lines forming an angle θ. Prove: let l be a constructible straight line, A a constructible point on l, and θ a constructible angle. The straight line(s) that go through A and form an angle θ with l is(are) constructible.

Problem Set 12 Due 04/27/2018 (complete)

1. Let F/K be a field extension, $S\subseteq T\subseteq F$ with S algebraically independent over K, and F algebraic over $K(T)$. Prove that there is a transcendence basis B, for F over K, such that $S\subseteq B\subseteq T$. (Hint: prove that a directed union of algebraically independent sets over K is algebraically

independent over K, and use Zorn's lemma)

2. Let F/K be a field extension and $S\subseteq F$. Prove that TFAE:
   I. S is maximal algebraically independent over K,
   II. S is algebraically independent over K and F is algebraic over $K(S)$,
   III. S is minimal such that F is algebraic over $K(S)$.
3. Let $F/E/K$ be a field tower. Prove that $$tr.d._K(F)=tr.d._E(F)+tr.d._K(E)$$
4. Let K be a field, and $t_1,\dots,t_n$ independent variables. If $f(t_1,\dots,t_n)\in K[t_1,\dots,t_n]$ is a symmetric polynomial in variables $t_1,\dots,t_n$, there is a **polynomial** g, such that $$ f(t_1,\dots,t_n) = g(s_1,\dots,s_n). $$ (Hint: Use double induction on n and d, the total degree of f)

Old Homework