

Math 401 - 01 Daily Topics - part 2 (Fall 2018)

$\newcommand{\aut}{\text{Aut}}$ $\newcommand{\inn}{\text{Inn}}$
 $\newcommand{\sub}{\text{Sub}}$ $\newcommand{\cl}{\text{cl}}$ $\newcommand{\join}{\vee}$
 $\newcommand{\bigjoin}{\bigvee}$ $\newcommand{\meet}{\wedge}$
 $\newcommand{\bigmeet}{\bigwedge}$ $\newcommand{\normaleq}{\unlhd}$
 $\newcommand{\normal}{\lhd}$ $\newcommand{\union}{\cup}$ $\newcommand{\intersection}{\cap}$
 $\newcommand{\bigunion}{\bigcup}$ $\newcommand{\bigintersection}{\bigcap}$ $\newcommand{\sq}[2][\sqrt{\#1\#2},\,]}{\sqrt{\#1\#2},\,}$ $\newcommand{\pbr}[1]{\langle \#1\rangle}$ $\newcommand{\ds}{\displaystyle}$
 $\newcommand{\C}{\mathbb{C}}$ $\newcommand{\R}{\mathbb{R}}$ $\newcommand{\Q}{\mathbb{Q}}$
 $\newcommand{\Z}{\mathbb{Z}}$ $\newcommand{\N}{\mathbb{N}}$ $\newcommand{\A}{\mathbb{A}}$
 $\newcommand{\F}{\mathbb{F}}$ $\newcommand{\T}{\mathbb{T}}$
 $\newcommand{\ol}[1]{\overline{\#1}}$ $\newcommand{\imp}{\rightarrow}$
 $\newcommand{\rimp}{\leftarrow}$ $\newcommand{\pinfty}{1/p^{\infty}}$
 $\newcommand{\power}{\mathcal{P}}$ $\newcommand{\calL}{\mathcal{L}}$
 $\newcommand{\calC}{\mathcal{C}}$ $\newcommand{\calN}{\mathcal{N}}$
 $\newcommand{\calB}{\mathcal{B}}$ $\newcommand{\calF}{\mathcal{F}}$
 $\newcommand{\calR}{\mathcal{R}}$ $\newcommand{\calS}{\mathcal{S}}$
 $\newcommand{\calU}{\mathcal{U}}$ $\newcommand{\calT}{\mathcal{T}}$
 $\newcommand{\gal}{\text{Gal}}$ $\newcommand{\isom}{\approx}$ $\newcommand{\idl}{\text{Idl}}$
 $\newcommand{\lub}{\text{lub}}$ $\newcommand{\glb}{\text{glb}}$ \$

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Week 7	Topics
10/01/2018	Test 1
10/02/2018	Lagrange's corollary 1
	Orbit-Stabilizer theorem
	Examples: cube, truncated icosahedron (soccer ball)
10/03/2018	Corollaries 2-5 to Lagrange's theorem
	Addendum to cor 3: moreover, there is a unique group of order p^2 , up to isomorphism.
	Thm. 7.2
	Example 6, p.144
	Corollary: if p is the smallest prime divisor of $ G $ and p^2 does not divide $ G $, then G has at most one subgroup of index p (HW)
10/05/2018	Thm. 7.3
Week 8	Topics
10/08/2018	Test 1 returned and reviewed
	Prop: if $\varphi: G \rightarrow H$ is an isomorphism, then so is $\varphi^{-1}: H \rightarrow G$.
	Prop: "isomorphic to" is an equivalence relation
	Thm. 6.1 Cayley's theorem
	$\text{Aut}(G)$, $\text{Inn}(G)$

10/09/2018	Thm 6.4 $\text{Aut}(G)$ is a group and $\text{Inn}(G)$ is a subgroup of $\text{Aut}(G)$
	Example: $\text{Inn}(D_4) \cong K_4$
	Prop: Let $G = \langle a \rangle$ cyclic and H a group
	1. A homom $\varphi: G \rightarrow H$ is uniquely determined by $\varphi(a)$.
	2. If G has order n and $u \in H$ has order d where $d n$, then there is (unique) homomorphism $\varphi: G \rightarrow H$ s.t. $\varphi(a) = u$. Moreover, φ is injective iff $d = n$.
	3. If G has infinite order and $u \in H$, then there is (unique) homomorphism $\varphi: G \rightarrow H$ s.t. $\varphi(a) = u$. Moreover, φ is injective iff u has infinite order.
	Example: $\text{Aut}(\mathbb{Z}_n) \cong U_n$
10/10/2018	Board presentations PS 6
	Thms. 10.2 and 6.3
10/12/2018	Fall break
Week 9	Topics
10/15/2018	Prop. Let $N \leq G$. TFAE
	(i) $gNg^{-1} \subseteq N$ for all $g \in G$
	(ii) $gNg^{-1} = N$ for all $g \in G$
	(iii) $gN = Ng$ for all $g \in G$
	(iv) the product of any two left cosets is a left coset.
	Moreover, in the last one, we have $(gN)(hN) = ghN$
	Def: normal subgroup
	Examples: 1. $A_n \trianglelefteq S_n$
	$\langle R_{360/n} \rangle \trianglelefteq D_n$
	Prop: if H is a subgroup of G of index 2, then H is a normal subgroup of G
	2. Prop: if $\varphi: G \rightarrow \bar{G}$ is a homomorphism, then $\ker(\varphi)$ is a normal subgroup of G
	3. If G is abelian, then every subgroup of G is normal
	4. $Z(G)$ is a normal subgroup of G .
	5. G and $\{1\}$ are normal subgroups of G .
	Thm 9.2 proof using (iv) above.
	Example 9.10 Generalize $\mathbb{Z}/n\mathbb{Z} \cong \mathbb{Z}_n$
10/16/2018	Example 9.12
	Thm 10.3 1st Isom Thm
	Example $\varphi: \mathbb{Z} \rightarrow \mathbb{Z}_n$
	Thm 9.4
	Thm (N/C theorem) Let $H \leq G$. $N_G(H) / C_G(H)$ is isomorphic to a subgroup of $\text{Aut}(H)$.
10/17/2018	proof of N/C theorem
	Example 10.17 $ G = 35$
	Thm 9.3
	Corollary: If $ G = pq$ and $Z(G) \neq 1$ then G is abelian.
	Thm 9.5 Cauchy's thm for abelian gps.
10/19/2018	Thm 10.4 $N \trianglelefteq G$, $\varphi: G \rightarrow G/N$ is an epimorphism with $\ker(\varphi) = N$

	Chapter 8 Direct Product
	Def: $G_1 \oplus G_2$
	Prop: 1) $G_1 \oplus G_2$ is a group.
	2) If G_1, G_2 are abelian then so is $G_1 \oplus G_2$.
	Examples: (1) $Z_2 \oplus Z_3$ is abelian of order 6, so it is isomorphic to Z_6
	(2) $G \oplus \{1\} \cong G \cong \{1\} \oplus G$
	Cor: $G_1 \oplus G_2$ contains subgroups isomorphic to G_1 and G_2 respectively.
	Def: $G_1 \oplus \dots \oplus G_n$
	Thm 8.1
Week 10	Topics
10/22/2018	Thm 8.2 G_1, G_2 finite. $G_1 \oplus G_2$ is cyclic iff G_1 and G_2 are cyclic or relatively prime orders.
10/23/2018	RSA cryptography. Public vs private keys
	Prop: $m^e \equiv m \pmod n$.
	Internal direct product
	Thm.: Let $H, K \leq G$ be such that $HK = G$ and $H \cap K = \{1\}$. Then $G \cong H \oplus K$.
	Def: When $H, K \leq G$ are such that $HK = G$ and $H \cap K = \{1\}$, we say that G is the internal direct product of H and K , and write $G = H \times K$.
	Example: Consider D_n with $n = 2m$ and m odd.
	Thm. 9.7 and corollary
	Prop: Let $H, N \leq G$.
	(1) If $N \triangleleft G$ then $HN \leq G$.
	(2) If $H, N \triangleleft G$ then $HN \triangleleft G$
10/24/2018	2nd, 3rd, 4th and 5th isomorphism theorems.
	$\text{sub}(D_4)$ and $\text{sub}(V_4)$ as examples.
10/26/2018	Thm If G is a finite abelian group of order n , and $m n$ then G has a subgroup of order m .
	Fund. Thm. of Finite Abelian Groups
	Statement and examples, $n=12$ and $n=600$
	Elementary divisors form, and invariant factors form
Week 11	Topics
10/29/2018	Board presentations. Problems sets 7 and 8
10/30/2018	Ch.24 Def: conjugate, conjugate class $\text{cl}(a)$.
	Prop: (1) "conjugate to" is an equivalence relation. The equivalence classes are the conjugacy classes.
	(2) $\text{cl}(a) = \{a\}$ iff $a \in Z(G)$
	Thm. 24.1 without finite assumption
	Cor. 1
	Thm. Class equation (2 versions)
	Thm. 24.2 A non-trivial p -group has non-trivial center.
	Def: Finite p -group. Metabelian group.
	Cor. Let p be a prime. If $ G = p^2$, then G is abelian.

	Cor. Let p be a prime. If $ G =p^3$, then G is metabelian. Moreover, $ Z(G) =p$ or $ Z(G) =p^3$.
	Example: Heisenber group H has order p^3 , and is not abelian.
10/31/2018	Thm. 24.3 Sylow's 1st Theorem
	Cor. Cauchy's theorem
	Cor. If $ G =pq$ where $p<q$ are primes and $p \nmid (q-1)$, then G is cyclic.
	Lemma 1. (1) Let $H \leq G$ and $C = \{gHg^{-1} \mid g \in G\}$ the set of all conjugates of H . Then $ C = [G:N_G(H)]$.
	Definition of Sylow p -subgroup.
	(2) Let $H, K \leq G$. If $HK = KH$ then $HK \leq G$.
	Lemma 2. Let P be a Sylow p -subgroup G . If $g \in N_G(P)$ and $ g $ is a power of p , then $g \in P$.
	Lemma 3. Let $ G =p^k m$ and $p \nmid m$. Let P be a Sylow p -subgroup of G , i.e. $ P =p^k$, and $H \leq G$ with $ H =p^l$ for some $l \leq k$. Then there is a conjugate of P that contains H , i.e. there is $g \in G$ such that $H \leq gPg^{-1}$.
11/02/2018	Board presentations. Problems set 9
	Proof of Lemma3
Week 12	Topics
11/05/2018	Sylow Theorems
	Examples: (1) $ G =35$ (2) $ G =455$ (3) $ G =21$ (4) $ G =256$
11/06/2018	Test 2
11/07/2018	Rings. Definitions: ring, unity, ring with unity (unitary ring), commutative ring, units of a unitary ring
	Examples
	Prop: The units of a ring, $U(R)$ form a multiplicative group.
11/09/2018	No class.

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