Math 330 - 01 Homework (Spring 2022)

- LaTeX-ed solutions are encouraged and appreciated.
- If you use LaTeX, hand-in a printed version of your homework.
- You are encouraged to discuss homework problems with classmates, but such discussions should NOT include the exchange of any written material.
- Writing of homework problems should be done on an individual basis.
- References to results from the textbook and/or class notes should be included.
- The following lists should be considered partial and tentative lists until the word complete appears next to it.
- Use 8.5in x 11in paper with smooth borders. Write your **name** on top of **each page**. Staple all pages.

$\newcommand{\aut}{\textrm{Aut}} \newcommand{\sub}{\textrm{Sub}} \newcommand{\join}{\vee} \newcommand{\bigjoin}{\bigvee} \newcommand{\meet}{\wedge} \newcommand{\bigmeet}{\bigwedge} \newcommand{\normaleq}{\unlhd} \newcommand{\normal}{\lhd} \newcommand{\union}{\cup} \newcommand{\intersection}{\cap} \newcommand{\bigunion}{\bigcup} \newcommand{\bigintersection}{\bigcap} \newcommand{\sq}[2][\ ]{\sqrt[#1]{#2\,}} \newcommand{\pbr}[1]{\langle #1\rangle} \newcommand{\ds}{\displaystyle} \newcommand{\C}{\mathbb{C}} \newcommand{\R}{\mathbb{R}} \newcommand{\Q}{\mathbb{Q}} \newcommand{\Z}{\mathbb{Z}} \newcommand{\N}{\mathbb{N}} \newcommand{\A}{\mathbb{A}} \newcommand{\F}{\mathbb{F}} \newcommand{\T}{\mathbb{T}} \newcommand{\ol}[1]{\overline{#1}} \newcommand{\imp}{\Rightarrow} \newcommand{\rimp}{\Leftarrow} \newcommand{\pinfty}{1/p^\infty} \newcommand{\power}{\mathcal{P}} \newcommand{\calL}{\mathcal{L}} \newcommand{\calC}{\mathcal{C}} \newcommand{\calN}{\mathcal{N}} \newcommand{\calB}{\mathcal{B}} \newcommand{\calF}{\mathcal{F}} \newcommand{\calR}{\mathcal{R}} \newcommand{\calS}{\mathcal{S}} \newcommand{\calU}{\mathcal{U}} \newcommand{\calT}{\mathcal{T}} \newcommand{\gal}{\textrm{Gal}} \newcommand{\isom}{\approx} \newcommand{\glb}{\textrm{glb}} $

Previous Homework

Problem Set 09 (complete) Due: 04/04/2022. Board presentation: 04/08/2022

1. Prove Prop. 8.40.ii
2. Prove Prop. 8.50
3. Give examples of subsets of $\R$ which are:
   I. bounded below and above,
   II. bounded below, but not bounded above,
   III. bounded above, but not bounded below,
   IV. not bounded above or below.
4. Project 9.3

Problem Set 08 (complete) Due: 03/28/2022. Board presentation: 04/01/2022

1. Prove Prop. 6.17
2. Prove Prop. 6.25 (first part)
3. Use Euclid's Lemma to prove the following corollary. Let p be a prime, $k\in\N$, $m_1,m_2,\dots,m_k\in\N$. If $p|(m_1m_2\cdots m_k)$ then there is some i with $1\leq i \leq k$ such that $p|m_i$. (Hint: Use induction on

k).
4. Prove Prop. 6.28

Problem Set 07 (complete) Due: 03/21/2022. Board presentation: 03/25-28/2022

1. Prove that set union is associative.
2. Prove Prop. 5.20.ii
3. Let X and Y be sets. Let $\power(X)$ denote the power set of X. Prove that: \[X\subseteq Y \iff \power(X)\subseteq\power(Y).\]
4. Let A be a set, and $\sim$ an equivalence relation on A. Let $(A/{\sim})$ be the partition consisting of all equivalence classes of $\sim$. Let $\Theta_{(A/{\sim})}$ be the equivalence relation induced by the partition $(A/{\sim})$. Prove that the two equivalence relations $\Theta_{(A/{\sim})}$ and $\sim$ are equal.

Problem Set 06 (complete) Due: 03/07/2022. Board presentation: 03/11/2022

1. Prove that $$\sum_{k=2}^n \binom{k}{2} = \binom{n+1}{3}$$ Hint: use induction on n.
2. Prove that for $k\geq 1$, $$\sum_{m=0}^k (-1)^m \binom{k}{m} = 0$$
3. Determine the base case, and prove by induction and using the recursive definition of Fibonacci numbers that $$f_{2k}=f_{k+1}^2-f_{k-1}^2$$
4. Prove Prop. 4.31 without using Prop. 4.29

Problem Set 05 (complete) Due: 02/28/2022. Board presentation: 03/04/2022

1. Prove Prop. 4.6.iii
2. Prove Prop. 4.11.ii
3. Prove Prop. 4.15.i
4. Prove Prop. 4.18

Problem Set 04 (complete) Due: 02/21/2022. Board presentation: 02/28/2022

1. Prove Prop. 2.38 (appendix)
2. Prove Prop. 2.41.iii (appendix)
3. Project 3.1

Problem Set 03 (complete) Due: 02/14/2022. Board presentation: 02/18/2022

1. Prove that for all $k\in\N$, k^2+k is divisible by 2. Is this true for all $k\in\Z$?
2. Prove Prop. 2.18.iii
3. Prove Prop. 2.21. Hint: use proof by contradiction.
4. Prove Prop. 2.23. Show, by counterexample, that the statement is not true if the hypothesis $m,n\in\N$ is removed.
5. Fill-in the blank and prove that for all $k\geq\underline{\ \ }$, $k^2 < 2^k$.

Problem Set 02 (complete) Due:02/07/2022. Board presentation: 02/11/2022

1. Prove Prop. 1.24
2. Prove Prop. 1.27.ii,iv
3. Prove Prop. 2.7.i,ii

4. Prove Prop. 2.12.iii

Problem Set 01 (complete) Due: 01/31/2022. Board presentation: 02/04/2022 (rescheduled for 02/07/2022)

1. Prove Prop. 1.7
2. Prove Prop. 1.11.iv
3. Prove Prop. 1.14
4. Prove that $1 + 1 \neq 1$. (Hint: assume otherwise, and get a contradiction).
   Can you prove that $1 + 1 \neq 0$?

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