Math 330 - 03 Homework (Fall 2018)

- LaTeX-ed solutions are encouraged and appreciated.
- If you use LaTeX, hand-in a printed version of your homework.
- You are encouraged to discuss homework problems with classmates, but such discussions should NOT include the exchange of any written material.
- Writing of homework problems should be done on an individual basis.
- References to results from the textbook and/or class notes should be included.
- The following lists should be considered partial and tentative lists until the word complete appears next to it.
- Use 8.5in x 11in paper with smooth borders. Write your **name** on top of **each page**. Staple all pages.

$\newcommand{\aut}{\textrm{Aut}} \newcommand{\sub}{\textrm{Sub}} \newcommand{\join}{\vee} \newcommand{\bigjoin}{\bigvee} \newcommand{\meet}{\wedge} \newcommand{\bigmeet}{\bigwedge} \newcommand{\normaleq}{\unlhd} \newcommand{\normal}{\lhd} \newcommand{\union}{\cup} \newcommand{\intersection}{\cap} \newcommand{\bigunion}{\bigcup} \newcommand{\bigintersection}{\bigcap} \newcommand{\sq}[2][\ ]{\sqrt[#1]{#2\,}} \newcommand{\pbr}[1]{\langle #1\rangle} \newcommand{\ds}{\displaystyle} \newcommand{\C}{\mathbb{C}} \newcommand{\R}{\mathbb{R}} \newcommand{\Q}{\mathbb{Q}} \newcommand{\Z}{\mathbb{Z}} \newcommand{\N}{\mathbb{N}} \newcommand{\A}{\mathbb{A}} \newcommand{\F}{\mathbb{F}} \newcommand{\T}{\mathbb{T}} \newcommand{\ol}[1]{\overline{#1}} \newcommand{\imp}{\Rightarrow} \newcommand{\rimp}{\Leftarrow} \newcommand{\pinfty}{1/p^\infty} \newcommand{\power}{\mathcal{P}} \newcommand{\calL}{\mathcal{L}} \newcommand{\calC}{\mathcal{C}} \newcommand{\calN}{\mathcal{N}} \newcommand{\calB}{\mathcal{B}} \newcommand{\calF}{\mathcal{F}} \newcommand{\calR}{\mathcal{R}} \newcommand{\calS}{\mathcal{S}} \newcommand{\calU}{\mathcal{U}} \newcommand{\calT}{\mathcal{T}} \newcommand{\gal}{\textrm{Gal}} \newcommand{\isom}{\approx} \newcommand{\glb}{\textrm{glb}} $

Problem Set 11 (complete) Due: 11/12/2018. Board presentation: 11/16/2018

1. Prove the following corollary to Prop. 10.4
   Corollary: $\glb(\R^+)=0$.
2. Prove Prop. 10.7
3. Prove Prop. 10.10.iii
4. Prove Prop. 10.13.ii

Problem Set 10 (complete) Due: 11/05/2018. Board presentation: 11/14/2018

1. Let $f:A\to B$ and $g:B\to C$ be functions.
   1. If $g\circ f$ is injective, then f is injective.
   2. If $g\circ f$ is surjective, then g is surjective.

2. Construct examples of functions $f:A\to B$ and $g:B\to C$ such that:
   1. $g\circ f$ is injective, but g is not injective.
   2. $g\circ f$ is surjective, but f is not surjective.
3. Prove Prop. 9.15 (Hint: induction)
4. Prove Prop. 9.18

Problem Set 09 (complete) Due: 10/29/2018. Board presentation: 11/05/2018

1. Prove Prop. 8.40.ii
2. Prove Prop. 8.41
3. Prove Prop. 8.50
4. Give examples of subsets of $\R$ which are:
   1. bounded below and above,
   2. bounded below, but not bounded above,
   3. bounded above, but not bounded below,
   4. not bounded above or below.

Problem Set 08 (complete) Due: 10/22/2018. Board presentation: 10/31/2018

1. Prove Prop. 6.16
2. Prove Prop. 6.17
3. Prove Prop. 6.25 (first part)
4. Use Euclid's Lemma to prove the following corollary. Let p be a prime, $k\in\N$, $m_1,m_2,\dots,m_k\in\N$. If $p|(m_1m_2\cdots m_k)$ then there is some i with $1\leq i \leq k$ such that $p|m_i$. (Hint: Use induction on k).

Problem Set 07 (complete) Due: 10/15/2018. Board presentation: 10/31/2018

1. Let A be a set, and $\sim$ an equivalence relation on A. Let $A/\sim$ be the partition consisting of all equivalence classes of $\sim$. Let $\Theta_{(A/\sim)}$ be the equivalence relation induced by the partition $A/\sim$. Prove that $\Theta_{(A/\sim)}=\ \sim$.
2. Do Project 6.8.iv.

Problem Set 06 (complete) Due: 10/08/2018. Board presentation: 10/31/2018

1. Prove that set union is associative.
2. Show, by counterexample, that set difference is not associative.
3. Prove Prop. 5.20.ii
4. Let X and Y be sets. Let $\power(X)$ denote the power set of X. Prove that: \[X\subseteq Y \iff \power(X)\subseteq\power(Y).\]
5. (challenge) Prove that symmetric difference is associative.

Problem Set 05 (complete) Due: 10/01/2018. Board presentation: 10/05/2018

1. Prove Prop. 4.6.iii
2. Prove Prop. 4.11.ii
3. Prove Prop. 4.15.i
4. Prove Prop. 4.16.ii

Problem Set 04 (complete) Due: 09/17/2018. Board presentation: 09/21/2018

1. Prove Prop. 2.38 (appendix)
2. Prove Prop. 2.41.iii (appendix)

Problem Set 03 (complete) Due: 09/12/2018. Board presentation: 09/17/2018

1. Prove that for all $k\in\N$, k^2+k is divisible by 2.
2. Prove Prop. 2.18.iii
3. Prove Prop. 2.21. Hint: use proof by contradiction.
4. Prove Prop. 2.23. Show, by counterexample, that the statement is not true if the hypothesis $m,n\in\N$ is removed.
5. Fill-in the blank and prove that for all $k\geq\underline{\ \ }$, $k^2 < 2^k$.

Problem Set 02 (complete) Due:09/05/2018. Board presentation: 09/10/2018

1. Prove Prop. 1.24
2. Prove Prop. 1.27.ii,iv
3. Prove Prop. 2.7.i,ii
4. Prove Prop. 2.12.iii

Problem Set 01 (complete) Due: 08/27/2018. Board presentation: 08/31/2018

1. Prove Prop. 1.7
2. Prove Prop. 1.11.iv
3. Prove Prop. 1.14
4. Prove that $1 + 1 \neq 1$. (Hint: assume otherwise, and get a contradiction).
   Can you prove that $1 + 1 \neq 0$?

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