Math 330 - 02 Previous Homework

- LaTeX-ed solutions are encouraged and appreciated.
- If you use LaTeX, hand-in a printed version of your homework.
- You are encouraged to discuss homework problems with classmates, but such discussions should NOT include the exchange of any written material.
- Writing of homework problems should be done on an individual basis.
- References to results from the textbook and/or class notes should be included.
- The following lists should be considered partial and tentative lists until the word complete appears next to it.
- Use 8.5in x 11in paper with smooth borders. Write your **name** on top of **each page**. Staple all pages.

$\newcommand{\aut}{\textrm{Aut}} \newcommand{\sub}{\textrm{Sub}} \newcommand{\join}{\vee} \newcommand{\bigjoin}{\bigvee} \newcommand{\meet}{\wedge} \newcommand{\bigmeet}{\bigwedge} \newcommand{\normaleq}{\unlhd} \newcommand{\normal}{\lhd} \newcommand{\union}{\cup} \newcommand{\intersection}{\cap} \newcommand{\bigunion}{\bigcup} \newcommand{\bigintersection}{\bigcap} \newcommand{\sq}[2][\ ]{\sqrt[#1]{#2\,}} \newcommand{\pbr}[1]{\langle #1\rangle} \newcommand{\ds}{\displaystyle} \newcommand{\C}{\mathbb{C}} \newcommand{\R}{\mathbb{R}} \newcommand{\Q}{\mathbb{Q}} \newcommand{\Z}{\mathbb{Z}} \newcommand{\N}{\mathbb{N}} \newcommand{\A}{\mathbb{A}} \newcommand{\F}{\mathbb{F}} \newcommand{\T}{\mathbb{T}} \newcommand{\ol}[1]{\overline{#1}} \newcommand{\imp}{\Rightarrow} \newcommand{\rimp}{\Leftarrow} \newcommand{\pinfty}{1/p^\infty} \newcommand{\power}{\mathcal{P}} \newcommand{\calL}{\mathcal{L}} \newcommand{\calC}{\mathcal{C}} \newcommand{\calN}{\mathcal{N}} \newcommand{\calB}{\mathcal{B}} \newcommand{\calF}{\mathcal{F}} \newcommand{\calR}{\mathcal{R}} \newcommand{\calS}{\mathcal{S}} \newcommand{\calU}{\mathcal{U}} \newcommand{\calT}{\mathcal{T}} \newcommand{\gal}{\textrm{Gal}} \newcommand{\isom}{\approx} \newcommand{\glb}{\textrm{glb}} $

Problem Set 10 (complete) Due: 11/17/2017. Board Presentation: 11/17/2017

1. Prove Prop. 10.23.v
2. Prove The. 10.26
3. Let (a_n) be a sequence. Consider the sequence of even-indexed terms, (a_{2n}), and the sequence of odd-indexed terms, (a_{2n+1}). Prove that if both (a_{2n}) and (a_{2n+1}) converge to L, then (a_n) converges to L.
4. Let $q_n=\displaystyle\frac{f_n}{f_{n+1}}$, where f_n is the n-th Fibonacci number. Show that the sequence (q_n) converges. What value does it converge to?

Problem Set 9 (complete) Due: 11/10/2017. Board presentation: 11/10/2017

1. Prove Prop. 10.10.iii
2. Prove Prop. 10.17
3. Prove Prop. 10.23.iii

Problem Set 8 (complete) Due: 11/03/2017. Board presentation: 11/03/2017

1. Prove Prop. 8.50
2. Prove that function composition is associative, when defined.

3. Let A,B,C be sets and $f:A\to B$ and $g:B\to C$ functions. Prove that if $g\circ f$ is surjective then g is surjective. Give an example when $g\circ f$ is surjective, but f is not.
4. Construct an example of a function with several right inverses.
5. Prove Prop. 9.15 (Hint: induction on k)
6. Prove Prop. 9.18

Problem Set 7 (complete) Due: 10/27/2017. Board presentation: 10/27/2017

1. Prove the corollary to Prop. 6.25: Let $a,b\in\Z$, $n\in\N$ and $k\geq 0$. If $a \equiv b \pmod{n}$ then $a^k \equiv b^k \pmod{n}$. (Hint: induction on k)
2. Prove Prop. 8.6
3. Prove Prop. 8.40.ii
4. Prove Prop. 8.41

Problem Set 6 (complete) Due: 10/13/2017. Board presentation: 10/20/2017

1. Let f_n be the n-th Fibonacci number. Prove by induction on n that \[\sum_{j=1}^n f_{2j} = f_{2n+1}-1 \]
2. Find and write down all the partitions on a 4-element set $A=\{a,b,c,d\}$. How many equivalence relations are there on A?
3. Prove Prop. 6.15
4. Prove Prop. 6.16

Problem Set 5 (complete) Due: 10/06/2017. Board presentation: 10/18/2017

1. Let $n\in\N$. Prove that if n is divisible by 3, then f_n is even. Is the converse true? If so, prove it; if not, give a counterexample.
2. Let $n\in\N$. Prove that if n is divisible by 5, then f_n is divisible by 5. Is the converse true? If so, prove it; if not, give a counterexample.
3. Prove the following identities for the Fibonacci numbers \[f_{2n+1}=f_n^2+f_{n+1}^2;\quad \\ f_{2n}=f_{n+1}^2-f_{n-1}^2 = f_n(f_{n+1}+f_{n-1}) \]
4. Prove the associativity of the set union and set intersection operations. Give a counterexample to show that set difference is not associative.

Problem Set 4 (complete) Due: 09/29/2017. Board presentation: 10/06/2017

1. Prove Prop. 4.6.iii
2. Prove Prop. 4.11.ii
3. Do project 4.12
4. Prove Prop. 4.16.ii

Problem Set 3 (complete) Due: 09/15/2017. Board presentation: 09/20/2017

1. Prove Prop. 2.21 (Hint: proof by contradiction)
2. Prove Prop. 2.23. Show, by counterexample, that the statement is not true when the hypothesis $m,n\in\N$ is removed.
3. Prove Prop. 2.38 (appendix)
4. Prove Prop. 2.41.iii (appendix)

Problem Set 2 (complete) Due: 09/08/2017. Board presentation: 09/15/2017

1. Prove Prop. 1.25
2. Prove Prop. 1.27.iv
3. Prove Prop. 2.7
4. Prove transitivity of $"\leq"$.

Problem Set 1 (complete) Due: 09/01/2017. Board Presentation: 09/08/2017

1. Prove Prop. 1.7
2. Prove that $1 + 1 \neq 1$. (Hint: assume otherwise, and get a contradiction). Can you prove that $1 + 1 \neq 0$?
3. Prove Prop. 1.11.iv
4. Prove Prop. 1.14

Current Homework

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