Math 330 - 02 Homework (Fall 2017)

- LaTeX-ed solutions are encouraged and appreciated.
- If you use LaTeX, hand-in a printed version of your homework.
- You are encouraged to discuss homework problems with classmates, but such discussions should NOT include the exchange of any written material.
- Writing of homework problems should be done on an individual basis.
- References to results from the textbook and/or class notes should be included.
- The following lists should be considered partial and tentative lists until the word complete appears next to it.
- Use 8.5in x 11in paper with smooth borders. Write your **name** on top of **each page**. Staple all pages.

$\newcommand{\aut}{\textrm{Aut}} \newcommand{\sub}{\textrm{Sub}} \newcommand{\join}{\vee} \newcommand{\bigjoin}{\bigvee} \newcommand{\meet}{\wedge} \newcommand{\bigmeet}{\bigwedge} \newcommand{\normaleq}{\unlhd} \newcommand{\normal}{\lhd} \newcommand{\union}{\cup} \newcommand{\intersection}{\cap} \newcommand{\bigunion}{\bigcup} \newcommand{\bigintersection}{\bigcap} \newcommand{\sq}[2][\ ]{\sqrt[#1]{#2\,}} \newcommand{\pbr}[1]{\langle #1\rangle} \newcommand{\ds}{\displaystyle} \newcommand{\C}{\mathbb{C}} \newcommand{\R}{\mathbb{R}} \newcommand{\Q}{\mathbb{Q}} \newcommand{\Z}{\mathbb{Z}} \newcommand{\N}{\mathbb{N}} \newcommand{\A}{\mathbb{A}} \newcommand{\F}{\mathbb{F}} \newcommand{\T}{\mathbb{T}} \newcommand{\ol}[1]{\overline{#1}} \newcommand{\imp}{\Rightarrow} \newcommand{\rimp}{\Leftarrow} \newcommand{\pinfty}{1/p^\infty} \newcommand{\power}{\mathcal{P}} \newcommand{\calL}{\mathcal{L}} \newcommand{\calC}{\mathcal{C}} \newcommand{\calN}{\mathcal{N}} \newcommand{\calB}{\mathcal{B}} \newcommand{\calF}{\mathcal{F}} \newcommand{\calR}{\mathcal{R}} \newcommand{\calS}{\mathcal{S}} \newcommand{\calU}{\mathcal{U}} \newcommand{\calT}{\mathcal{T}} \newcommand{\gal}{\textrm{Gal}} \newcommand{\isom}{\approx} \newcommand{\glb}{\textrm{glb}} $

Problem Set 12 (complete) Due: 12/08/2017. Board presentation: 12/08/2017

1. Prove that if A and B are finite sets, then $A\union B$ is a finite set.
2. Prove the following corollary to Proposition 13.6.
   I. If $f:A\to B$ is injective and B is finite, then A is finite.
   II. If $g:A\to B$ is surjective and A is finite, then B is finite.
3. Do Project 13.15, finding a formula for the bijection in the picture.
4. Prove Theorem 13.28.

Problem Set 11 (complete) Due: 12/01/2017. Board Presentation: 12/01/2017

1. Write down the details of the proofs that the sum of a rational number and an irrational number is irrational, and that the product of a non-zero rational number and an irrational number is irrational.
2. Prove the converse of Prop. 11.2
3. Do Project 11.14
4. Prove that for all $x,y,z,w\in\R$ with $z,w\neq 0$,
   $$\frac{x}{z}+\frac{y}{w}=\frac{xw+yz}{zw}\qquad\textrm{and}\qquad\frac{x}{z}\frac{y}{w}=\frac{xy}{zw}$$
5. Consider the set $$A=\{x\in\Q\mid x^2<2\}$$ Show that A is non-empty and has an upper bound in $\Q$, but

does not have a least upper bound in $\Q$. Hint: by way of contradiction, assume A has a least upper bound u in $\Q$, and compare it with $\sqrt{2}$.

6. Consider the sequence defined recursively by $$a_n=a_{n-1}+3a_{n-2} \\ a_1=1 \\ a_2=2.$$ Use the converse of Proposition 11.25 to find a closed formula for a_n.

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