

TeX code compiled with `\documentclass{beamer}` using the Amsterdam theme.

There is one png image needed to compile slides:

problem2.png

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\begin{document} \begin{frame} \begin{block}{Steps to Solve a Related Rate} \begin{itemize} \item[\bf (i)] What
is variable? \item[\bf (ii)] Which rates are known? \item[\bf (iii)] Which rates need to be found?
\item[\bf (iv)] What equation relates the variables in (ii)? \item[\bf (v)] Use Implicit
Differentiation on the equation in (iii) to relate the rates. \end{itemize} \end{block} If $V$ is the volume of a cube
with edge lengths $x$ and the cube expands as time passes, find $\frac{dV}{dt}$ in terms of $\frac{dx}{dt}$.
\begin{itemize} \item[\bf a)] What is $\frac{dV}{dx}$ when $x=4$ inches and is growing at a rate of $2$ inches
per minute? \item[\bf b)] What is $x$ if the volume is shrinking at $3$ cubic inches per minute and the side length
is shrinking at $4$ inches per minute? \item[\bf c)] Can a cube have a shrinking volume and a growing sides?
\end{itemize} \end{frame} \begin{frame} \large A spherical weather balloon is being inflated at a rate of $(0.5
m^3/sec)$. \begin{enumerate}[a)] \item How fast is the diameter increasing at the instant the
diameter is 2 meters? \item How fast is the volume changing at that same instant? \item
How fast is the surface area changing at that same instant? \end{enumerate} \end{frame} \begin{frame} \vskip
10pt As gravel is being poured into a conical pile, its volume $V$ changes with time. As a result, the height $h$ and
radius $r$ also change with time. Knowing that at any moment $V = \frac{1}{3}\pi r^2 h$, the relationship between
the changes with respect to time in the volume, radius and height is \begin{enumerate} \item
$\displaystyle \frac{dV}{dt} = \frac{1}{3}\pi (2r \frac{dr}{dt} h + r^2 \frac{dh}{dt})$ \item
$\displaystyle \frac{dV}{dt} = \frac{1}{3}\pi (2r \frac{dr}{dt} \cdot \frac{dh}{dt})$ \item
$\displaystyle \frac{dV}{dr} = \frac{1}{3}\pi (2rh + r^2 \frac{dh}{dt})$ \item
$\displaystyle \frac{dV}{dh} = \frac{1}{3}\pi (r^2 + 2r \frac{dr}{dh} h)$ \end{enumerate}
\end{frame} \begin{frame} \large Imagine the following magic triangle. Its base is on a horizontal surface and no
matter what you do to its height, the triangle always has area $10$ cm$^2$. \vskip 20pt \large If you push down
on the top of the triangle so that it becomes shorter at a rate of $3$ cm/sec, how fast will the length of the base be
changing when the triangle is $5$ cm tall? \end{frame} \begin{frame} \large My neighbors have a very loud
stereo. The volume knob turns half a circle (angles $\theta$ between $0^\circ$ and $180^\circ$) and the volume of
the music is given by the function $V(\theta) = 110 \sin(\theta/2)$ decibels (dB). \vskip 20pt One night at $3:30$ in
the morning I notice an increase from a volume of $88$ dB at a rate of $1$ decibel per second! At what rate can I
deduce that my neighbor is turning his volume knob? \end{frame} \begin{frame} %%% Students should
discover errors in this "solution" \small Water is leaking out of a tank shaped like a right circular cone with height
$5$ m and top radius $3$ m. When the water level in the cone is $2$ m, the water level is decreasing at a rate of
$0.1 \frac{m}{s}$. How fast is the water leaking out of the cone? \pause \begin{columns}
\begin{column}{0.45\textwidth} \begin{center} \includegraphics[width=5cm]{problem2.png} \end{center}
\end{column} \vskip 10pt \begin{column}{0.55\textwidth} \small The volume of the water in the cone is $V =
\frac{1}{3}\pi r^2 h$ and using the figure above and similar triangles $\frac{r}{h} = \frac{3}{5}$, which
means $r = \frac{3}{5}h = \frac{3}{5} \cdot 2 = \frac{6}{5}$. This means that $V = \frac{1}{3}\pi (\frac{6}{5})^2 h =
\frac{12\pi}{25} h$. Taking the derivative with respect to time $\frac{dV}{dt} = \frac{12\pi}{25} \frac{dh}{dt} =
\frac{12\pi}{25} \cdot 10 = \frac{6\pi}{25} \frac{m^3}{s}$. \end{column} \end{columns} \begin{frame} \small Water
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\includegraphics[width=5cm]{problem2.png} \end{center} \end{column} \begin{column}{0.55\textwidth} \small
The volume of the water in the cone is $V = \frac{1}{3}\pi r^2 h$ and using the figure above and similar triangles
$\frac{r}{h} = \frac{3}{5}$, which means $r = \frac{3}{5}h$. This means that $V = \frac{1}{3}\pi (\frac{3}{5}h)^2 h =
\frac{1}{3}\pi (\frac{3}{5})^2 h^3 = \frac{12\pi}{25} h^3$. Taking the derivative with respect to time $\frac{dV}{dt} =
\frac{12\pi}{25} \cdot 3h^2 \frac{dh}{dt} = \frac{36\pi}{25} h^2 \frac{dh}{dt} = \frac{36\pi}{25} (2)^2 \frac{dh}{dt} =
\frac{144\pi}{25} \frac{dh}{dt}$. \end{column} \end{columns}

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= $\frac{-36\pi}{250} \frac{m^3}{s}$

1. A streetlight hangs 5 meters above the ground. Regina, who is 1.5 meters tall, walks away from the point under the light at a rate of 2 meters per second. How fast is her shadow lengthening when she is 7 meters away from the point under the light? (Hint: Use similar triangles.)

2. Suppose Regina has the ability to magically shrink herself. At what rate must she do this to keep her shadow a constant length of 3 meters? Write this as a function of only her distance from the point under the light.

3. A revolving beacon from a light house shines on the straight shore, and the closest point on the shore is a pier one half mile from the lighthouse. Let θ denote the angle between the lighthouse, pier, and point on the shore where the light shines.

4. Write the distance from the pier to the point of light as a function of θ .

5. What is the rate of change of the distance from the pier to the point of light with respect to θ .

6. Suppose θ is a function of time t . Give an expression for the rate of change of distance with respect to time t .

7. Suppose that the light makes 1 revolution per minute. How fast is the light traveling along the straight beach at the instant it passes over a shorepoint 1 mile away from the shorepoint nearest the searchlight?

8. Given that a spherical raindrop evaporates at a rate proportional to its surface area, how fast does the radius shrink?

9. The minute hand on a watch is 8 mm long and the hour hand is 4 mm long. How fast is the distance between the tips of the hands changing at one o'clock?

10. Find the shortest line segment with endpoints on the x and y axes going through the point $(1, 8)$.

11. What is the area of the triangle formed by the shortest line segment?

12. What is the rate of change of area with respect to the x -coordinate of the point on the x -axis?

13. For which x is the area increasing?

14. The speed limit on a stretch of highway is 55 mph. Highway patrol officer, Sgt. Miguel, stations himself at a point, out of view of the motorists, 50 feet off the highway. Miguel is equipped with a radar gun which measures the speed at which a car approaches his position.

15. He takes a reading of suspected speeders by pointing his radar gun at a point on the highway 120 feet from the point on the highway closest to him. The radar gun picks up a reading of 48 feet/sec for a green Chevy driven by Alyssa. How fast is she traveling? Is Alyssa speeding?

16. At a certain moment, ship A is 6 miles south and 8 miles west of ship B. Ship A at that moment is steaming east at 12 mph, while ship B is steaming north at 15 mph.

17. Are the ships approaching each other or separating from each other? At what rate?

18. Particle A moves along the positive horizontal axis, and particle B along the graph of $f(x) = -\sqrt{3x}$, $x \leq 0$. At a certain time, A is at the point $(5, 0)$ and moving with speed 3 units/sec; and B is at a distance of 3 units from the origin moving with speed 4 units/sec. At what rate is the distance between A and B changing?

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Permanent link: https://www2.math.binghamton.edu/p/calculus/resources/calculus_flipped_resources/derivatives/2.8_related_rates_text

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