

TeX code compiled with `\documentclass{beamer}` using the Amsterdam theme.
There is one png image needed to compile slides:

[problem2.png](#)

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\begin{document} \begin{frame} \begin{block}{Steps to Solve a Related Rate} \begin{itemize}
\item[\bf (i)] What is variable? \ What is constant? \vskip 2pt \item[\bf (ii)] Which rates are known? \
Which rates need to be found? \vskip 2pt \item[\bf (iii)] What equation relates the variables in (ii)? \vskip
2pt \item[\bf (iv)] Use Implicit Differentiation on the equation in (iii) to relate the rates. \end{itemize}
\end{block} If $V$ is the volume of a cube with edge lengths $x$ and the cube expands as time passes,
find $\frac{dV}{dt}$ in terms of $\frac{dx}{dt}$. \begin{itemize} \item[\bf a)] What is $\frac{dV}{dx}$
when $x=4$ inches and is growing at a rate of $2$ inches per minute? \item[\bf b)] What is $x$ if the
volume is shrinking at $3$ cubic inches per minute and the side length is shrinking at $4$ inches per
minute? \item[\bf c)] Can a cube have a shrinking volume and a growing sides? \end{itemize}
\end{frame} \begin{frame} \large A spherical weather balloon is being inflated at a rate of \ ( 0.5
m^{\{3\}}/sec \). \vskip 5pt \begin{enumerate}[a)] \item How fast is the diameter increasing at the instant
the diameter is 2 meters? \vskip 15pt \item How fast is the volume changing at that same instant? \vskip
15pt \item How fast is the surface area changing at that same instant? \end{enumerate} \end{frame}
\begin{frame} \vskip 10pt As gravel is being poured into a conical pile, its volume $V$ changes with
time. As a result, the height $h$ and radius $r$ also change with time. Knowing that at any moment
$V=\frac{1}{3}\pi r^2 h$, the relationship between the changes with respect to time in the volume,
radius and height is \vskip 10pt \begin{enumerate} \item $\displaystyle{\frac{dV}{dt}=\frac{1}{3}\pi
\left( 2r\frac{dr}{dt} h+r^2\frac{dh}{dt}\right)}$ \item $\displaystyle{\frac{dV}{dt}=\frac{1}{3}\pi
\left( 2r\frac{dr}{dt} \cdot \frac{dh}{dt}\right)}$ \item $\displaystyle{\frac{dV}{dr}=\frac{1}{3}\pi
\left( (r^2)(1)+2r\frac{dr}{dh}h\right)}$ \end{enumerate} \end{frame} \begin{frame} {\large Imagine the
following magic triangle. Its base is on a horizontal surface and no matter what you do to its height, the
triangle always has area $10$ cm$^2$.} \vskip 20pt {\large If you push down on the top of the triangle
so that it becomes shorter at a rate of $3$ cm/sec, how fast will the length of the base be changing when
the triangle is $5$ cm tall?} \end{frame} \begin{frame} \large My neighbors have a very loud stereo.
The volume knob turns half a circle (angles $\theta$ between $0^\circ$ and $180^\circ$) and the
volume of the music is given by the function $V(\theta)=110\sin(\theta/2)$ decibels (dB). \vskip 20pt One
night at $3:30$ in the morning I notice an increase from a volume of $88$ dB at a rate of $1$ decibel per
second! At what rate can I deduce that my neighbor is turning his volume knob? \end{frame}
\begin{frame} %%%%%%%%% Students should discover errors in this "solution" {\small Water is leaking
out of a tank shaped like a right circular cone with height $5$ m and top radius $3$ m. When the water
level in the cone is $2$ m, the water level is decreasing at a rate of $0.1 \frac{m}{s}$. How fast is the
water leaking out of the cone?} \pause \begin{columns} \begin{column}{0.45\textwidth}
\begin{center} \includegraphics[width=5cm]{problem2.png} \end{center} \end{column} \pause
\begin{column}{0.55\textwidth} {\small The volume of the water in the cone is $V = \frac{1}{3}\pi
r^2 h$ and using the figure above and similar triangles $\frac{r}{h} = \frac{3}{5}$ \pause , which
means} $$ {\small $r = \frac{3}{5}h = \frac{3}{5} \cdot 2 = \frac{6}{5}$.} $$ {\small This means that} $$
{\small $V = \frac{1}{3}\pi \left(\frac{6}{5}\right)^2 \pi h = \frac{12\pi}{25}h$.} $$ \end{column}
\end{columns} \pause {\small Taking the derivative with respect to time} $$ {\small $\frac{dV}{dt} =
\frac{12\pi}{25}\frac{dh}{dt} = \frac{12\pi}{25}\frac{1}{10} = \frac{6\pi}{125} \frac{m^3}{s}$.} $$
\end{frame} \begin{frame} {\small Water is leaking out of a tank shaped like a right circular cone with
height $5$ m and top radius $3$ m. When the water level in the cone is $2$ m, the water level is
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decreasing at a rate of $\$0.1 \frac{m}{s}$. How fast is the water leaking out of the cone?

$\begin{columns} \begin{column}{0.45\textwidth} \begin{center}$

$\includegraphics[width=5cm]{problem2.png} \end{center} \end{column}$

$\begin{column}{0.55\textwidth} \{ \small \text{The volume of the water in the cone is } V = \frac{1}{3} \pi r^2 h \text{ and using the figure above and similar triangles } \frac{r}{h} = \frac{3}{5}, \text{ which means } r = \frac{3}{5} h \} \} \{ \small \text{This means that } V = \frac{1}{3} \pi \left(\frac{3}{5} h \right)^2 h = \frac{3 \pi}{25} h^3 \} \} \{ \small \text{Taking the derivative with respect to time } \frac{dV}{dt} = \frac{3 \pi}{25} 3 h^2 \frac{dh}{dt} = \frac{3 \pi}{25} 3 (2)^2 \frac{dh}{dt} = \frac{-36 \pi}{25} \frac{m^3}{s} \} \} \end{column}$

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From:

<http://www2.math.binghamton.edu/> - **Department of Mathematics and Statistics, Binghamton University**

Permanent link:

http://www2.math.binghamton.edu/p/calculus/resources/calculus_flipped_resources/derivatives/2.8_related_rates_text 

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